Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE/ *NASIONALE SENIORSERTIFIKAAT*

GRADE/*GRAAD 12*

SEPTEMBER 2023

MATHEMATICS P1/*WISKUNDE V1* MARKING GUIDELINE/*NASIENRIGLYN*

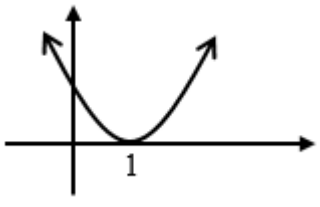
MARKS/*PUNTE:* **150**

This marking guideline consists of 17 pages./
Hierdie nasienriglyn bestaan uit 17 bladsye.

NOTE/*LET WEL*:

- If a candidate answers a question TWICE, mark the FIRST attempt ONLY.
  Indien 'n kandidaat 'n vraag TWEE keer beantwoord, merk SLEGS die EERSTE poging.
- Consistent accuracy applies in ALL aspects of the marking guideline.
  Volgehoue akkuraatheid geld deurgaans in ALLE aspekte van die nasienriglyn.
- If a candidate crossed out an attempt of a question and did not redo the question, mark the crossed-out attempt.
  Indien 'n kandidaat 'n poging vir 'n vraag deurgetrek het en nie die vraag weer beantwoord het nie, merk die poging wat deurgetrek is.
- The mark for substitution is awarded for substitution into the correct formula.
  Die punt vir substitusie word toegeken vir substitusie in die korrekte formule.

QUESTION 1/*VRAAG 1*

1.1.1	$x^2+x-30=0$ $(x-5)(x+6)=0$ $\therefore x=5$ or / *of* $x=-6$ **OR/*OF*** $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $=\frac{-1\pm\sqrt{(1)^2-4(1)(-30)}}{2(1)}$ $=\frac{-1\pm\sqrt{121}}{2}$ $=5$ or / *of* -6	✓ factors / *faktore* ✓ $x=5$ ✓ $x=-6$ (3) **OR/*OF*** ✓ substitution / *vervanging* ✓ $x=5$ ✓ $x=-6$ (3)
1.1.2	$x(2x-6)=-3$ $2x^2-6x+3=0$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $=\frac{-(-6)\pm\sqrt{(-6)^2-4(2)(3)}}{2(2)}$ $=\frac{6\pm\sqrt{12}}{4}$ **OR / *OF*** $\frac{3\pm\sqrt{3}}{2}$ $=2,37$ or / *of* $0,63$	✓ standard form / *standaardvorm* ✓ substitution / *vervanging* ✓ $x=2,37$ or/*of* ✓ $x=0,63$ (4)
1.1.3	$x^2-2x+1>0$ $(x-1)(x-1)>0$ $c/v: x=1$ $\therefore x\in\mathbb{R}, x\neq 1$  	✓ factors / *faktore* ✓✓$x\in\mathbb{R}, x\neq 1$ (Accuracy/*Akkuraatheid*) (3)

1.1.4	$2x-1=\sqrt{4-5x}$ $(2x-1)^2=\left(\sqrt{4-5x}\right)^2$ $(2x-1)^2=4-5x$ $4x^2-4x+1+5x-4=0$ $4x^2+x-3=0$ $(4x-3)(x+1)=0$ or / *of* $x=\frac{-1\pm\sqrt{(1)^2-4(4)(-3)}}{2(4)}$ $\therefore x=\frac{3}{4}$ or / *of* $x\neq -1$	✓ squaring both sides *kwadreer beide kante* ✓ standard form / *standaardvorm* ✓ factors / *formula* *faktore* / *formule* ✓ answers with selection *antwoorde met seleksie/keuse* (4)
1.2	$y-2x=-1$.....(1) $y^2+2xy=3x^2$.....(2) $y=2x-1$.....(3) Substitute / *Vervang* (3) into/*in* (2): $(2x-1)^2+2x(2x-1)-3x^2=0$ $4x^2-4x+1+4x^2-2x-3x^2=0$ $5x^2-6x+1=0$ $(5x-1)(x-1)=0$ $\therefore x=\frac{1}{5}$ or / *of* $x=1$ $\therefore y=-\frac{3}{5}$ or / *of* $y=1$ **OR/*OF*** $y-2x=-1$.....(1) $y^2+2xy=3x^2$.....(2) $x=\frac{y+1}{2}$.....(3) Substitute / *Vervang* (3) into / *in* (2) $y^2+2y\left(\frac{y+1}{2}\right)-3\left(\frac{y+1}{2}\right)^2=0$ $y^2+y^2+y-3\left(\frac{y^2+2y+1}{4}\right)=0$ $8y^2+4y-3y^2-6y-3=0$ $5y^2-2y-3=0$ $(5y+3)(y-1)=0$ $\therefore y=-\frac{3}{5}$ or / *of* $y=1$ $\therefore x=\frac{1}{5}$ or / *of* $x=1$	✓ $y=2x-1$ ✓ substitution / *vervanging* ✓ standard form / *standaardvorm* ✓ factors / *faktore* ✓ *x*-values / *x-waardes* ✓ *y*-values / *y-waardes* (6) **OR/*OF*** ✓ $x=\frac{y+1}{2}$ ✓ substitution / *vervanging* ✓ standard form / *standaardvorm* ✓ factors / *faktore* ✓ *y*-values / *y-waardes* ✓ *x*-values / *x-waardes* (6)

1.3	$2x^2 - px + 1 = 0$ For real unequal roots: *Vir ongelykereële wortels* : $b^2 - 4ac > 0$ $(-p)^2 - 4(2)(1) > 0$ $p^2 - 8 > 0$ $\therefore\ p < -\sqrt{8} \quad or / of \quad p > -\sqrt{8}$	✓ $b^2 - 4ac > 0$ ✓ substitution / *vervanging* ✓ standard form / *standaardvorm* ✓ ✓ answer / *antwoord* (5)
		[25]

QUESTION 2/*VRAAG 2*

2.1.1	$a + 9d = 21$ $a + 16d = 49$ $\therefore -7d = -28$ $d = 4$	✓ $a + 9d = 21$ ✓ $a + 16d = 49$ ✓ value of d / *waarde van d* (3)
2.1.2	$a + 9(4) = 21$ $a = -15$ $T_{18} = T_{17} + 4$ $= 49 + 4$ $= 53$ $\therefore\ T_1 + T_{18}$ $= -15 + 53$ $= 38$	✓ $a = -15$ ✓ $T_{18} = 53$ ✓ answer / *antwoord* (3)
2.2.1	$T_1 = 4(1) - 19 = -15$ $T_2 = 4(2) - 19 = -11$ $T_3 = 4(3) - 19 = -7$	✓ all three terms / *al drie terme* (1)
2.2.2	$S_n = \frac{n}{2}[2a + (n-1)d]$ $S_m = \frac{m}{2}[2(-15) + 4(m-1)]$ $1189 = \frac{m}{2}(-30 + 4m - 4)$ $0 = 2m^2 - 17m - 1189$ $(2m + 41)(m - 29) = 0 \qquad$ or / *of* $m = \frac{-(-17) \pm \sqrt{(-17)^2 - 4(2)(-1189)}}{2(2)}$ $\therefore m = 29 \quad$ or / *of* $\quad m \neq -\frac{41}{2}$	✓ substitution / *vervanging* and/en = 1 189 ✓ standard form / *standaardvorm* ✓ method / *metode* ✓ answer / *antwoord* (4)

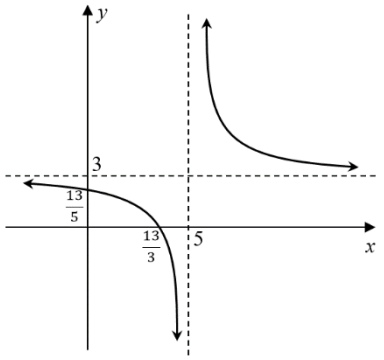
2.3.1	$-78 \quad -76 \quad -72 \quad -66$ $2 \quad 4 \quad 6$ $2 \quad 2$ $-58\ ;\ -48$	✓ both answers / *beide antwoorde* (1)
2.3.2	$T_n = an^2 + bn + c$ $2a = 2 \quad 3a + b = 2 \quad a + b + c = -78$ $a = 1 \quad 3(1) + b = 2 \quad 1 - 1 + c = -78$ $b = -1 \quad c = -78$ $\therefore T_n = n^2 - n - 78$	✓ $a = 1$ ✓ $b = -1$ ✓ $c = -78$ ✓ answer / *antwoord* (4)
2.3.3	$k > 78$ (Accept/*Aanvaar* $k \geq 78$)	✓✓ answer / *antwoord* (2)
		[18]

QUESTION 3/*VRAAG 3*

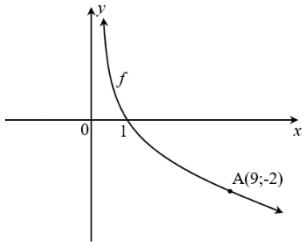
3.1	$a = 81$ .....(1) $a + ar^2 = 117$ .....(2) $a\left(1 + r^2\right) = 117$ $81\left(1 + r^2\right) = 117$ $1 + r^2 = \frac{117}{81}$ $r^2 = \frac{36}{81}$ $r = \pm\frac{2}{3}$	✓ ✓ $a + ar^2 = 117$ ✓ substitution / *vervanging* ✓ simplification / *vereenvoudiging* ✓ answer / *antwoord* (4)
3.2.1	$r = \frac{9^x}{3^x}$ $= \frac{3^{2x}}{3^x}$ $= 3^x$	✓ answer / *antwoord* (3)

3.2.2	$S_\infty = \frac{a}{1-r}$ $\frac{1}{2} = \frac{3^x}{1-3^x}$ $2.3^x = 1-3^x$ $3.3^x = 1$ $3^x = \frac{1}{3}$ $3^x = 3^{-1}$ $\therefore\ x = -1$	✓ $a = 3^x$ & $r = 3^x$ ✓ substitution / *vervanging* ✓ answer / *antwoord* (3)
		[8]

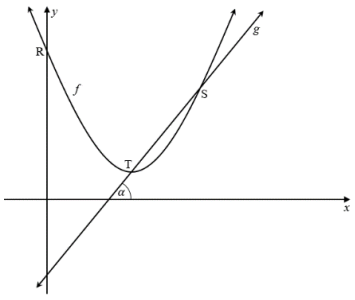
QUESTION 4/*VRAAG 4*

4.1	$f(x)=\frac{2}{x-5}+3$ $x=5$ $y=3$	✓ $x=5$ ✓ $y=3$ (2)
4.2	$y \in \mathbb{R}$ but/*maar* $y \neq 3$	✓ $y \neq 3$ (1)
4.3	$f(x)=\frac{2}{x-5}+3$ $x-$intercept / $x-$*afsnit*: $\frac{2}{x-5}+3=0$ $\frac{2}{x-5}=-3$ $-3x+15=2$ $x=\frac{13}{3}$ $y-$intercept / $y-$*afsnit*: $y=\frac{2}{0-5}+3$ $=\frac{13}{5}$ $\therefore$ Intercepts/*Afsnitte*: $(\frac{13}{3};0)$ and / *en* $\left(0;\frac{13}{5}\right)$	✓ substitution / *vervanging* ✓ $x=\frac{13}{3}$ ✓ $y=\frac{13}{5}$ (3)
4.4		✓ asymptotes / *asimptote* ✓ *y*-intercept / *y-afsnit* ✓ *x*-intercept / *x-afsnit* ✓ shape and quadrants *vorm* en *kwadrante* (4)
4.5	f is reflected in the *x*-axis and shifted 2 units downwards. f *is gereflekteer in die x-as en* 2 *eenhede afwaarts geskuif.* **OR/*OF*** f is shifted 2 units upwards and then reflected in the *x*-axis. f *is* 2 *eenhede opwaarts geskuif en daarna gereflekteer in die x-as.*	✓ $f(x)$ reflected / *gereflekteer* ✓ in the *x*-axis / *in die x-as* ✓ shift 2 units / *skuif 2 eenhede* downwards/upwards *afwaarts/opwaarts* (3)
	–	**[13]**

QUESTION 5/*VRAAG 5*

		
5.1	$f(x) = \log_b x$ $x = b^y$ $9 = b^{-2}$ $b^2 = \frac{1}{9}$ $b = \frac{1}{3}$	✓ substitution / *vervanging* ✓ answer / *antwoord* (2)
5.2	$y = \log_{\frac{1}{3}} x$ $x = \log_{\frac{1}{3}} y$ $y = \left(\frac{1}{3}\right)^x$ **OR/*OF*** $y = 3^{-x}$	✓ swopping x and y *omruil van x en y* ✓ answer / *antwoord* (2)
5.3	$0 < x \leq 1$	✓ ✓ answer (Accuracy) *antwoord* (*Akkuraatheid*) (2)
5.4	$y = 0$	✓ ✓ answer (Accuracy) *antwoord* (*Akkuraatheid*) (2)
		[8]

QUESTION 6/*VRAAG 6*

		
6.1	$f(x) = x^2 - 6x + 11$ $= x^2 - 6x + 9 - 9 + 11$ $= (x-3)^2 + 2$ $\therefore$ At TP : $x = 3$ and / *en* $y = 2$ **OR/*OF*** $f(x) = x^2 - 6x + 11$ $x = -\frac{b}{2a} = -\frac{(-6)}{2(1)}$ $= 3$ $\therefore y = 3^2 - 6(3) + 11$ $= 2$ $\therefore$ At TP : $x = 3$ and / *en* $y = 2$	✓ completing the square *vierkantsvoltooiing* ✓ $(x-3)^2 + 2$ ✓✓ values for x and y *waardes van x en y* ✓ substitution / *vervanging* ✓ value of x / *waarde van x* ✓ substitution / *vervanging* ✓ value of y / *waarde van y* (4)
6.2	$m_g = \tan 63,44^\circ$ $= 2$ $y - 2 = 2(x-3)$ $y = 2x - 4$	✓ $m_g = 2$ ✓ substitution / *vervanging* ✓ equation of g / *vergelyking van g* (3)
6.3	$f(x) = g(x)$ $x^2 - 6x + 11 = 2x - 4$ $x^2 - 8x + 15 = 0$ $(x-3)(x-5) = 0$ $x = 3$ or / *of* $x = 5$ $\therefore y = 2(5) - 4$ $= 6$ $\therefore S(5\ ;\ 6)$ CA only if g is linear *VA slegs as g linieêr is*	✓ equating / *gelyk stel* ✓ standard form / *standaardvorm* ✓ x values / *x-waardes* ✓ S coordinates / S-*koördinate* (4)

6.4.1	$1 \le x \le 5$	✓✓ answer / *antwoord* (2)
6.4.2	$k \le -2$ Accept / *Aanvaar* $k < -2$ for 1 mark / *vir* 1 *punt*	✓✓ answer / *antwoord* (2)
		[15]

QUESTION 7/*VRAAG 7*

7.1	$A = P(1+i)^n$ $166\,433 = 97\,000\left(1+\frac{0,091}{4}\right)^{4n}$ $\frac{166\ 433}{97\ 000} = \left(\frac{4\ 091}{4\ 000}\right)^n$ $\therefore 4n = \log_{\frac{4091}{4000}} \frac{166\ 433}{97\ 000}$ $= 24$ $\therefore n = 6$ *years* / *jaar*	✓ $\frac{0,091}{4}$ ✓ substitution into correct formula *vervanging in korrekte formule* ✓ correct use of logs *korrekte gebruik van logs* ✓ answer / *antwoord* (4)
7.2.1	$A = P(1-i)^n$ $= 482000(1-0,147)^5$ $= R217666,80$	✓ substitution into correct formula *vervanging in korrekte formule* ✓ answer / *antwoord* (2)
7.2.2	$A = P(1+i)^n$ $= 482\,000(1+0,081)^5$ $= \text{R}\,711500,99$	✓ substitution into correct formula *vervanging in korrekte formule* ✓ answer / *antwoord* (2)

7.2.3	Required amount / *Bedrag benodig*: $= \text{R}711\ 501 - \text{R}217\,666,80$ $= \text{R}493\,834,20$ $F = \frac{x[(1+i)^n - 1]}{i}$ $493\,834,20 = \frac{x\left[\left(1+\frac{0,073}{12}\right)^{60} - 1\right]}{\frac{0,073}{12}}\left(1+\frac{0,073}{12}\right)$ $\therefore x = \frac{493\,834,20 \times \frac{0,073}{12}}{\left[\left(1+\frac{0,073}{12}\right)^{60} - 1\right]\left(1+\frac{0.073}{12}\right)}$ $= \text{R}6\,803,01$	✓ amount / *bedrag* ✓ correct formula / *korrekte formule* ✓ $n = 60$ *and / en* $i = \frac{0,073}{12}$ ✓ $\frac{x\left[\left(1+\frac{0,073}{12}\right)^{60} - 1\right]}{\frac{0,073}{12}}$ ✓ $\times\left(1+\frac{0,073}{12}\right)$ ✓ answer / *antwoord* (6)
		[14]

QUESTION 8/*VRAAG 8*

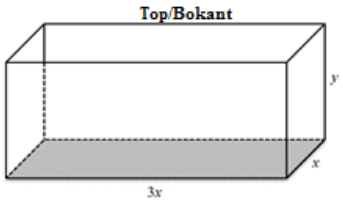
8.1	$f(x)=1-x^2$ $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $=\lim\limits_{h\to 0}\dfrac{1-(x+h)^2-(1-x^2)}{h}$ $=\lim\limits_{h\to 0}\dfrac{1-x^2-2xh-h^2-1+x^2}{h}$ $=\lim\limits_{h\to 0}\dfrac{-2xh-h^2}{h}$ $=\lim\limits_{h\to 0}\dfrac{h(-2x-h)}{h}$ $=\lim\limits_{h\to 0}(-2x-h)$ $=-2x$ **OR/*OF*** $f(x)=1-x^2$ $f(x+h)-f(x)=1-(x+h)^2-(1-x^2)$ $=1-x^2-2xh-h^2-1+x^2$ $=-2xh-h^2$ $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $=\lim\limits_{h\to 0}\dfrac{-2xh-h^2}{h}$ $=\lim\limits_{h\to 0}\dfrac{h(-2x-h)}{h}$ $=\lim\limits_{h\to 0}(-2x-h)$ $=-2x$	✓ substitution / *vervanging* ✓ expansion / *uitbreiding* ✓ simplification / *vereenvoudiging* ✓ factorisation / *faktorisering* ✓ answer / *antwoord* (5) **OR/*OF*** ✓ substitution / *vervanging* ✓ expansion / *uitbreiding* ✓ simplification / *vereenvoudiging* ✓ factorisation / *faktorisering* ✓ answer / *antwoord* (5)
8.2.1	$D_x\left[\left(x-\dfrac{1}{x}\right)^2\right]=D_x\left(x^2+\dfrac{1}{x^2}-2\right)$ $=D_x\left(x^2+x^{-2}-2\right)$ $=2x-2x^{-3}$	✓ $D_x\left(x^2+x^{-2}-2\right)$ ✓ $2x$ *and / en* constant/*konstante* is 0 ✓ $-2x^{-3}$ (3)
8.2.2	$y=\dfrac{x^5}{10}-\dfrac{2}{\sqrt{x}}$ $=\dfrac{1}{10}x^5-2x^{-\frac{1}{2}}$ $\therefore \dfrac{dy}{dx}=\dfrac{1}{2}x^4+x^{-\frac{3}{2}}$	✓ $2x^{-\frac{1}{2}}$ ✓ $\frac{1}{2}x^4$ ✓ $x^{-\frac{3}{2}}$ (3)
		[11]

QUESTION 9/*VRAAG 9*

9.1	$f(x) = -2x^3 + ax^2 + bx + c.$ $9 = -2(2)^3 + a(2)^2 + b(2) - 3$ $9 = -16 + 4a + 2b - 3$ $4a + 2b = 28 \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$ At TP / *By* $DP: -6x^2 + 2ax + b = 0$ $-6(2)^2 + 2a(2) + b = 0$ $4a + b = 24 \ldots\ldots(2)$ $4a + 2b = 28$ $4a + b = 24$ (1) − (2) $b = 4$ $4a + 4 = 24$ $4a = 20$ $a = 5$	✓ equation 1 / *vergelyking* 1 ✓ $f'(x) = -6x^2 + 2ax + b$ ✓ equation 2 / *vergelyking* 2 ✓ value of b / *waarde van b* ✓ value of a / *waarde van a* (5)
9.2	$f(x) = -2x^3 + 5x^2 + 4x - 3$ $f'(x) = -6x^2 + 10x + 4$ At/*By* E: $-6x^2 + 10x + 4 = 0$ $3x^2 - 5x - 2 = 0$ $(3x + 1)(x - 2) = 0$ $\therefore x = -\frac{1}{3}$ or / *of* $x = 2$ $y = -2(-\frac{1}{3})^3 + 5(-\frac{1}{3})^2 + 4(-\frac{1}{3}) - 3$ $= -\frac{100}{27}$ $\therefore E\left(-\frac{1}{3}; -\frac{100}{27}\right)$	✓ $f'(x) = 0$ ✓ values of x / *waarde van x* ✓ both coordinates / *beide koördinate* (3)
9.3.1	$x < -\frac{1}{3}$ *or / of* $x > 2$	✓ $x < -\frac{1}{3}$ ✓ $x > 2$ (2)

9.3.2	$f''(x) = -12x + 10$ $-12x + 10 = 0$ $x = \frac{5}{6}$ $\therefore x > \frac{5}{6}$ **OR/*OF*** $x = -\frac{b}{3a}$ $= -\frac{5}{3(-2)}$ $= \frac{5}{6}$ $\therefore\ x > \frac{5}{6}$	✓ $f''(x) = -12x + 10$ ✓ value of x / *waarde van x* ✓ answer / *antwoord* (3) **OR/*OF*** ✓ substitution / *vervanging* ✓ value of x / *waarde van x* ✓ answer / *antwoord* (3)
9.4	$f'(x) = -6x^2 + 10x + 4$ $m = f'(-1) = -6(-1)^2 + 10(-1) + 4$ $= -12$ $\therefore y = -12x + c$ $0 = -12(-1) + c$ $c = -12$ $y = -12x - 12$	✓ $f'(x) = -6x^2 + 10x + 4$ ✓ m ✓ substitution / *vervanging* ✓ answer / *antwoord* (4)
		[17]

QUESTION 10/*VRAAG 10*

10.1	 $3x^2+2xy+6xy=147$ $3x^2+8xy=147$ $\therefore y=\dfrac{147-3x^2}{8x}.$	✓ $3x^2+2xy+6xy=147$ ✓ simplifying / *vereenvoudiging* (2)
10.2	$V=lbh$ $=3x.x.y$ $=3x^2\left(\dfrac{147-3x^2}{8x}\right)$ $=\dfrac{441x}{8}-\dfrac{9x^3}{8}$ $V'(x)=\dfrac{441}{8}-\dfrac{27x^2}{8}$ $\therefore \dfrac{441}{8}-\dfrac{27x^2}{8}=0$ $27x^2=441$ $x^2=\dfrac{441}{27}$ $x=\dfrac{21}{3\sqrt{3}}\ \ (=4,04)$	✓ $3x.x.y$ ✓ substitution / *vervanging* ✓ $V'(x)=0$ ✓ simplification / *vereenvoudiging* ✓ answer / *antwoord* (5)
		[7]

QUESTION 11/*VRAAG 11*

	WATCH SOCCER/ *KYK SOKKER*	**WATCH RUGBY/ *KYK RUGBY***	**TOTAL/ *TOTAAL***
Female / *Vroulik*	72	a	120
Male / *Manlik*	54	36	90
Total / *Totaal*	b	84	210

11.1.1

$a = 48$
$b = 126$

✓ $a = 48$
✓ $b = 126$

(2)

11.1.2

$P(F \text{ and/}en\ WS) = \frac{72}{210}$

✓✓ answer / *antwoord*

(2)

11.1.3

(For independent events) / (*Vir onafhanklike gebeurtenisse*)
$P(M) \times P(R) = P(M \text{ and / } en\ R)$

$P(M) \times P(R) = \frac{90}{210} \times \frac{84}{210}$
$= \frac{6}{35}$
$\square\ 0{,}17$

✓ $\frac{90}{210} \times \frac{84}{210}$

✓ answer / *antwoord*

$P(M \text{ and / } en\ R) = \frac{36}{210}$
$= \frac{6}{35}$

✓ $\frac{36}{210}$

$\therefore$ The events are independent
Die gebeurtenisse is onafhanklik

✓ conclusion / *gevolgtrekking*

(4)

11.2.1

26	25	24	10	9	8

26 x 25 x 24 x 10 x 9 x 8
= 11 232 000

✓ method / *metode*
✓ answer / *antwoord*

(2)

11.2.2	5 x 25 x 24 x 9 x 8 x 3 = 648 000 $P(\text{Vowel / Factor of 9}) / P(\textit{Vokaal / Faktor van 9})$ $= \frac{648\,000}{11232\ 000}$ $= \frac{3}{52}$	✓ $5\times25\times24$ ✓ $9\times8\times3$ ✓ 11232 000 as denominator / *as noemer* ✓ answer / *antwoord* (4)

TOTAL/*TOTAAL:* 150