Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE
NASIONALE SENIOR SERTIFIKAAT

GRADE/*GRAAD* 12

JUNE/*JUNIE* 2021

MATHEMATICS P2/*WISKUNDE V2*
MARKING GUIDELINE/*NASIENRIGLYN*
(EXEMPLAR/*EKSEMPLAAR)*

MARKS/*PUNTE*: 150

This marking guideline consists of 17 pages.
Hierdie nasienriglyn bestaan uit 17 bladsye.

QUESTION 1/*VRAAG 1*

1.1	$a = 62$ $b = 51$ $c = 42$ $d = 35$ $e = 31$	✓ value of a / *waarde van a* ✓ value of b / *waarde van b* ✓ value of c / *waarde van c* ✓ value of d / *waarde van d* ✓ value of e / *waarde van e* (5)
1.2	Skewed to the right / *Skeef na regs* **OR/*OF*** Positively skewed / *Positief skeef*	✓ answer / *antwoord* (1)
1.3	Yes / *Ja* $Q_3 = 51$ and the upper 25% is from Q_3 above. $Q_3 = 51$ *en die boonste* 25% *is vanaf* Q_3 *en op.*	✓ Yes / *Ja* ✓Reason / *Rede* (2)
		[8]

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QUESTION 2/*VRAAG 2*

2.1

Positive impact / *Positiewe impak*

The number of learners obtaining lower marks decreased while those obtaining higher marks increased in the Post Test.
Die aantal leerders wat laer punte behaal het, het verminder terwyl die wat hoër punte behaal het in die na-toets vermeerder het.

✓ Positive impact
Positiewe impak
✓ Reason / *Rede*

(2)

2.2

$20 < x \leq 30$

✓ answer / *antwoord* (1)

2.3

Less / *Minder*

✓answer / *antwoord* (1)

2.4

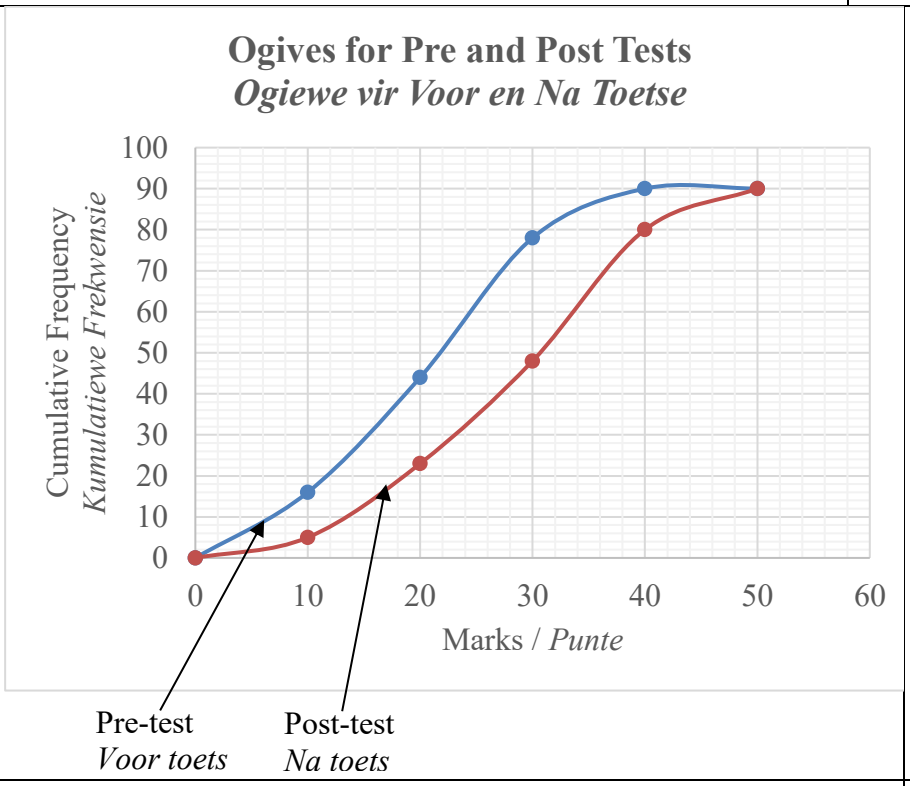
Marks ***Punte***	**Frequency** ***Frekwensie***		**Cumulative Frequency** ***Kumulatiewe Frekwensie***	
	Pre-Test ***Voor Toets***	**Post Test** ***Na Toets***	**Pre-Test** ***Voor Toets***	**Post Test** ***Na Toets***
$0 \leq x < 10$	16	5	16	5
$10 \leq x < 20$	28	18	44	23
$20 \leq x < 30$	34	25	78	48
$30 \leq x < 40$	12	32	90	80
$40 \leq x < 50$	0	10	90	90

Frequency / *Frekwensie*
✓ Pre-Test / *Voor Toets*
✓ Post Test / *Na Toets*

Cumulative Frequency
Kumulatiewe Frekwensie
✓ Pre-Test / *Voor Toets*
✓ Post Test / *Na Toets*

(4)

2.5



✓ grounding / *anker*

✓ upper limits used
boonste limiete

✓ shape / *vorm*

(3)

2.6

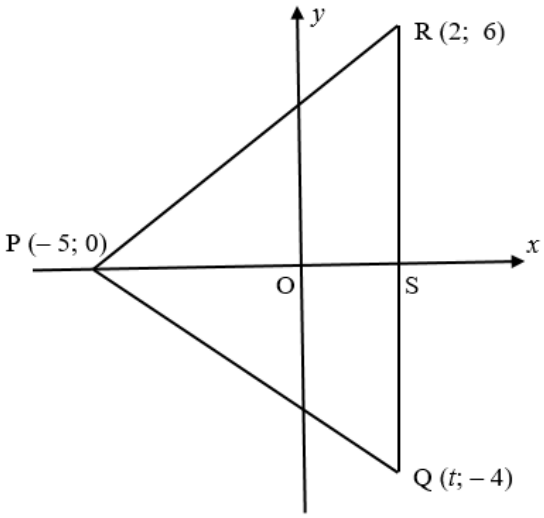
Pre: 90 – 78 = 12 learners obtained 60% and more
Post: 90 – 48 = 42 learners obtained 60% and more
Therefore, the teacher achieved the target.
Voor - toets: 90 – 78 = 12 *leerders het* 60% *en meer behaal*
Na - toets: 90 – 48 = 42 *leerders het* 60% *en meer behaal*
Daarom het die onderwyser die doelwit behaal

✓ 12
✓ 42
✓ conclusion
gevolgtrekking

(3)

[14]

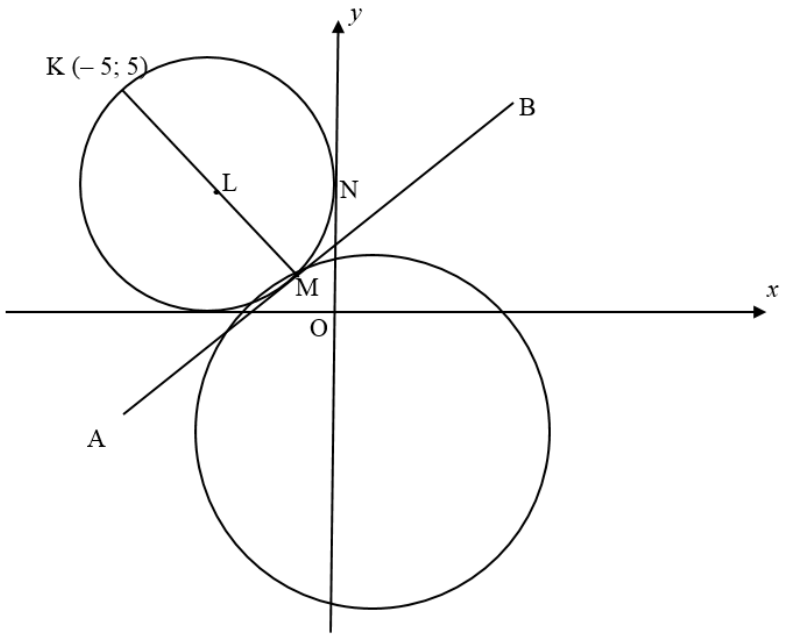
QUESTION 3/*VRAAG 3*



3.1	$t = 2$	✓ value of *t / waarde van t* (1)
3.2.1	$PR = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(-5-2)^2 + (0-6)^2}$ $= \sqrt{85}$	✓substitution / *vervanging* ✓answer / *antwoord* (2)
3.2.2	$m_{PR} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-6}{-5-2}$ $= \frac{6}{7}$	✓substitution / *vervanging* ✓gradient of PR *gradiënt van* PR (2)
3.3	$\tan \hat{RPS} = \frac{6}{7}$ $\hat{RPS} = 40,6°$ $40,6° + \hat{PRQ} = 90°$ (ext. $\angle$ of Δ / *buite* $\angle$ *van* Δ) $\therefore \hat{PRQ} = 49,4°$	✓ $\tan\theta = \frac{6}{7}$ ✓ value of / *waarde van* $\hat{PRQ}$ ✓ method / *metode* ✓ reason / *rede* ✓value of / *waarde van* $\hat{RPS}$ (5)
3.4	$m_{PR} = \frac{6}{7}$ $m_{PQ} = \frac{-4-0}{2-(-5)} = \frac{-4}{7}$ $m_{PR} \times m_{PQ} = \frac{6}{7} \times \left(-\frac{4}{7}\right) = -\frac{24}{49} \neq -1$ $\therefore \Delta PRQ$ is not right angled at R ΔPRQ *is nie reghoekig by R nie* **OR / *OF***	✓ substitution / *vervanging* ✓ gradient of PR / gradiënt van PR ✓ method / *metode* ✓ conclusion / *gevolgtrekking* **OR / *OF***

	$PQ=\sqrt{(-5-2)^2+(0-(-4))^2}$ $=\sqrt{65}$ $RQ^2=100$ $PR^2=85$ $PQ^2=65$ $\therefore RQ^2 \neq PR^2+PQ^2$ $\therefore \Delta PQR$ is not right angled at P ΔPRQ *is nie reghoekig by R nie*	✓ substitution / *vervanging* ✓ squares / *vierkante* ✓ $RQ^2 \neq RP^2+PQ^2$ ✓ conclusion / *gevolgtrekking* (4)
3.5	$m_{newline}=m_{PQ}=-\frac{4}{7}$ $y-0=-\frac{4}{7}(x-0)$ $\therefore y=-\frac{4}{7}x$	✓ gradient of new line *gradiënt van nuwe lyn* ✓ substitution / *vervanging* ✓equation / *vergelyking* (3)
3.6	SP = 7 units / *eenhede* SR = 6 units / *eenhede* RQ = 10 units / *eenhede* Area of ΔSPR / *Oppervlakte van* ΔSPR $=\frac{1}{2}\times 7\times 6$ $=21$ units^2 /eenhede^2 Area of ΔRPQ / *Oppervlakte van* ΔRPQ $=\frac{1}{2}\times 7\times 10$ $=35$ units^2 / *eenhede*^{2} $\frac{\text{Area of } \Delta SPR}{\text{Area of } \Delta PRQ} / \frac{Oppervlakte\,van\,\Delta SPR}{Oppervlakte\,van\,\Delta PRQ}$ $=\frac{21}{35}$ $=\frac{3}{5}$	✓ length of SP / *lengte van* SP ✓ length of RQ / *lengte van* RQ ✓ Area of /*Oppervlakte van* ΔSPR ✓ Area of /*Oppervlakte van* ΔRPQ ✓answer/*antwoord* (5)
		[22]

QUESTION 4/*VRAAG 4*



4.1.1	$x^2+y^2+6x-6y+9=0$ $x^2+6x+9+y^2-6y+9=-9+9+9$ $\therefore (x+3)^2+(y-3)^2=9$ $\mathrm{L}(-3;3)$ and/*en* $r=3$ units/*eenhede*	✓ method / *metode* ✓ $(x+3)^2+(y-3)^2=9$ ✓ coordinates of L *koördinate van* L ✓ value of *r* / *waarde van r* (4)
4.1.2	$-3=\frac{-5+x_M}{2}$ $3=\frac{5+y_M}{2}$ $x_M=-1$ $y_M=1$ $\therefore \mathrm{M}(-1;1)$	✓ method / *metode* ✓ value of *x* / *waarde van x* ✓ value of *y* / *waarde van y* (3)
4.1.3	$m_{KL}=\frac{5-1}{-5+1}=-1$ *OR* $m_{LM}=\frac{3-1}{-3+1}=-1$ $m_{\text{tangent}}=1$ $y-1=1(x-(-1))$ $\therefore y=x+2$	✓ m_{LM} *OR* m_{KM} ✓ m_{tangent} ✓ substitution / *vervanging* ✓ equation / *vergelyking* (4)
4.1.4	$(x+3)^2+(y-3)^2=9$ $(0+3)^2+(y-3)^2=9$ $(y-3)^2=0$ $y=3$ $\mathrm{N}(0;3)$	✓ value of *x* / *waarde van x* ✓ value of *y* / *waarde van y* (2)

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4.2.1	L (– 3 ; 3) L/ (2 ; –4)	✓ value of *x / waarde van x* ✓ value of *y / waarde van y* (2)
4.2.2	$m_{ML'} = \frac{-4-1}{2+1} = -\frac{5}{3}$ $y-(1) = -\frac{5}{3}(x+1)$ $\therefore y = -\frac{5}{3}x - \frac{2}{3}$ Not passing through the origin *Gaan nie deur die oorsprong nie*	✓ $m_{ML'}$ ✓substitution / *vervanging* ✓equation / *vergelyking* ✓ conclusion / *gevolgtrekking* (4)
		[19]

QUESTION/*VRAAG 5*

5.1.1	$\sin\alpha = -\frac{5}{13}$ and/*en* $\tan\beta = -\frac{3}{4}$ $x = -12$ $r = 5$ $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ $= \left(\frac{-5}{13}\right)\left(\frac{-4}{5}\right) + \left(\frac{-12}{13}\right)\left(\frac{3}{5}\right)$ $= -\frac{16}{65}$	✓ value of *x / waarde van x* ✓ value of *r / waarde van r* ✓ expansion / *uitbreiding* ✓ substitution / *vervanging* ✓ answer / *antwoord* (5)
5.1.2	$\cos 2\beta = 1 - 2\sin^2\beta$ $= 1 - 2\left(\frac{3}{5}\right)^2$ $= \frac{7}{25}$	✓ expansion / *uitbreiding* ✓ substitution / *vervanging* ✓ answer / *antwoord* (3)
5.1.3	$\tan(-\alpha - 180°) = -\tan(180° + \alpha)$ $= -\tan\alpha$ $= -\left(\frac{-5}{-12}\right) = -\frac{5}{12}$	✓ reduction / *reduksie* ✓ substitution / *vervanging* (2)
5.2.1	$1 - \cos\theta = 0$ *or* $\sin\theta = 0$ $\cos\theta = 1$ *or* $\sin\theta = 0$ $\therefore \theta = 180.k \quad (k \in$ **OR / *OF*** $\theta = 360°.k$ or $\theta = 180° + 360°.k \quad (k \in$	✓ method / *metode* ✓ answer / *antwoord* (2)
5.2.2	LHS/*LK* $= \frac{\sin\theta}{1-\cos\theta} - \frac{\cos\theta}{\sin\theta}$ $= \frac{\sin^2\theta - \cos(1-\cos\theta)}{\sin\theta(1-\cos\theta)}$ $= \frac{\sin^2\theta - \cos + \cos^2\theta}{\sin\theta(1-\cos\theta)}$ $= \frac{1-\cos\theta}{\sin\theta(1-\cos\theta)}$ $= \frac{1}{\sin\theta}$ $=$ RHS/*RK*	✓ common denominator *gemene noemer* ✓ simplification / *vereenvoudiging* ✓ identity */ identiteit* ✓ simplification / *vereenvoudiging* (4)

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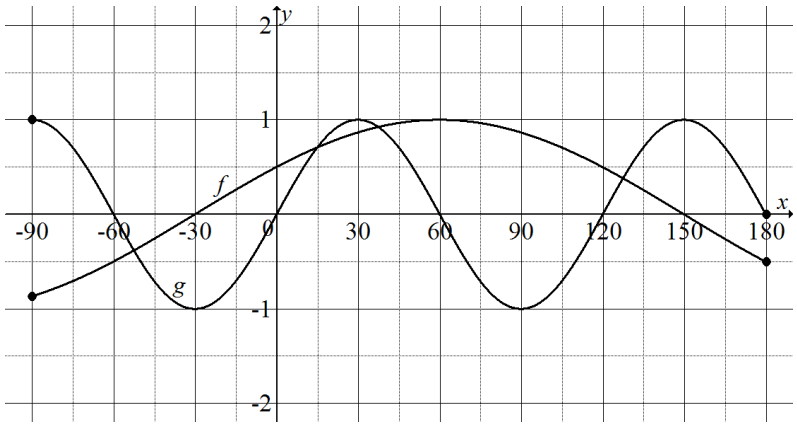
5.3	LHS/*LK* $= \frac{\sin(x-y)}{\cos x.\cos y}$ $= \frac{\sin x\cos y-\cos x\sin y}{\cos x.\cos y}$ $= \frac{\sin x\cos y}{\cos x.\cos y}-\frac{\cos x\sin y}{\cos x.\cos y}$ $= \tan x-\tan y$ $= 3k-2k$ $= k$	✓ identity / *identiteit* ✓method / *metode* ✓ simplification / *vereenvoudiging* ✓substitution / *vervanging* (4)
		[20]

QUESTION 6/*VRAAG 6*

6.1.1	1	✓ 1 (1)
6.1.2	120°	✓ 120° (1)
6.2	$f(x)=g(x)$ $\cos(x-60°)=\sin 3x$ $\cos(x-60°)=\cos(90°-3x)$ $\pm(x-60°)=90°-3x+k.360°\quad (k\in\mathbb{Z})$ $x-60°=90°-3x+k.360°$ or / *of* $-x+60°=90°-3x+k.360°$ $4x=150°+k.360°$ or / *of* $2x=30°+k.360°$ $x=37,5°+k.90°$ or / *of* $x=15°+k.180°$ $x\in\{37,5;-52,5°;15°;127,5°\}$	✓ $\cos(x-60°)=\cos(90°-3x)$ ✓ without cos *sonder* cos ✓ simplification *vereenvoudiging* ✓ $x=30°+k.90°$ *or / of* $x=15°+k.180°$ ✓ two values *twee waardes* ✓ two remaining values *twee oorblywende waardes* (6)
6.3		*f:* ✓ endpoints / *eindpunte* ✓ both intercepts *beide afsnitte* ✓ shape / *vorm* *g:* ✓ both intercepts *beide afsnitte* ✓ shape / *vorm* (5)
6.4	$x=-30°$ or/*of* $x=150°$	✓both values of x *beide waardes van x* (1)
6.5	$f(x)=\cos(x-60°+15°)$ $h(x) = \cos(x-45°)$	✓$h(x) = \cos(x-45°)$ (1)
		[15]

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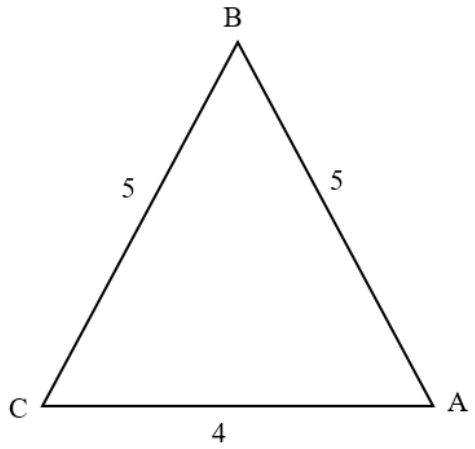
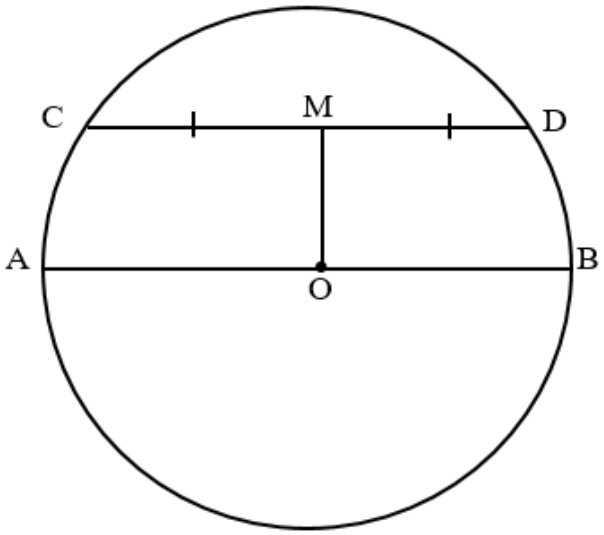
QUESTION 7/*VRAAG 7*



$5^2 = 4^2 + 5^2 - 2(4)(5)\cos A$ $\cos A = \frac{2}{5}$	✓ substitution into cosine rule / *vervanging in die cosinusreël* ✓ value of cos A / *waarde van* cos A
$4^2 = 5^2 + 5^2 - 2(5)(5)\cos B$ $\cos B = \frac{17}{25}$	✓ substitution into cosine rule / *vervanging in die cosinusreël* ✓ value of cos B / *waarde van* cos B
$\cos A - \cos B = \frac{2}{5} - \frac{17}{25}$ $= -\frac{7}{25} \approx -0,28$	✓ value of cos A – cos B / *waarde van* cos A – cos B (5)
	[5]

QUESTION 8/*VRAAG 8*



$OM \perp CD$ (line from centre which bisects the chord) (*lyn vanaf die middelpunt wat koord halveer*)	✓ S/R
$AO = OB = OD = 11\text{cm}$	✓ S
In ΔOMD: $MD^2 + 7^2 = 11^2$ (Pythagoras theorem/*stelling*) $MD = 6\sqrt{2}$ $\therefore CD = 12\sqrt{2}$	✓ use of Pythagoras theorem *gebruik van Pythagoras - stelling* ✓ length of MD/*lengte van* MD ✓ length of CD/*lengte van* CD (5)
	[5]

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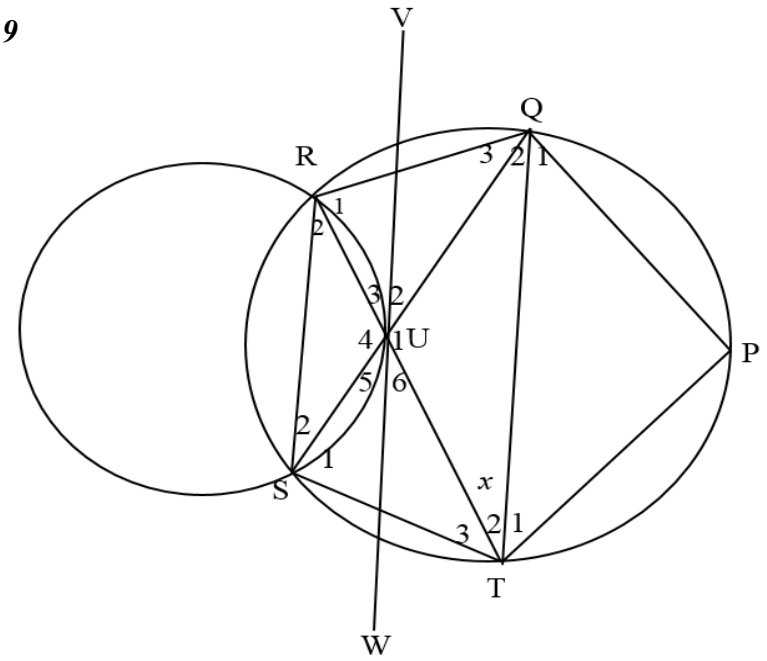
QUESTION 9/*VRAAG 9*



9.1	$\hat{R}_2 = \hat{Q}_2$ ($\angle$s in the same segment/*$\angle e$ in dieselfde segment*) $\hat{U}_4 = \hat{U}_1$ (vert. opp. $\angle$s / *regoorst. $\angle e$*) $\hat{S}_2 = \hat{T}_2$ ($\angle$s in the same segment / *$\angle e$ in dieselfde segment*) $\therefore \Delta RUS \,|||\, \Delta QUT$ ($\angle\angle\angle$)	✓ S/R ✓ S/R ✓ S/R (3)
9.2	$\hat{S}_2 = \hat{T}_2 = x$ ($\angle$s in the same seg / *$\angle e$ in dieselfde segment*) $\hat{U}_3 = \hat{S}_2 = x$ (tan-chord theorem / *raaklyn-koord stelling*) $\hat{U}_6 = \hat{U}_3 = x$ (vertically opp. $\angle$s / *regoorst. $\angle$e*)	✓ S/R ✓ S ✓ R ✓S/R (4)
9.3.1	$\hat{R}_1 + x + 90° - x = 180°$ (sum of $\angle$s of Δ / *som van die $\angle e$ van Δ*) $\therefore \hat{R}_1 = 90°$ QT is a diameter (QT subtends a right angle) QT *is 'n middellyn* (QT *onderspan 'n reghoek*)	✓ S ✓ value of $\hat{R}_1$ / *waarde van* $\hat{R}_1$ ✓ QT is a diameter/ *is 'n middellyn* ✓ R (4)
9.3.2	$\hat{P} = 90°$ ($\angle$in the semicircle / *$\angle$in 'n semisirkel*) **OR / OF** $\hat{P} + 90° = 180°$ (opp. $\angle$s of a cyclic quad / *teenoorst. $\angle e$ van 'n koordevierhoek*) $\therefore \hat{P} = 90°$	✓ S ✓ R (2)

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9.4.1	$\hat{Q}_2 = \hat{T}_2 = x$ (∠s opp. equal sides / *∠e teenoor gelyke sye*) $\therefore \hat{Q}_2 = \hat{S}_2$ $\therefore$ RS|| QT (Alt. ∠s are equal / *Verw. ∠e is gelyk*)	✓ S/R ✓ R (2)
9.4.2	$\hat{U}_2 = \hat{Q}_2 = x$ VW is a tangent to circle passing through QUT (Converse of tan-chord theorem) VW is 'n raaklyn aan die sirkel wat deur QUT gaan (*Omgekeerde van die raaklyn – koord stelling*)	✓S ✓R (2)
		[17]

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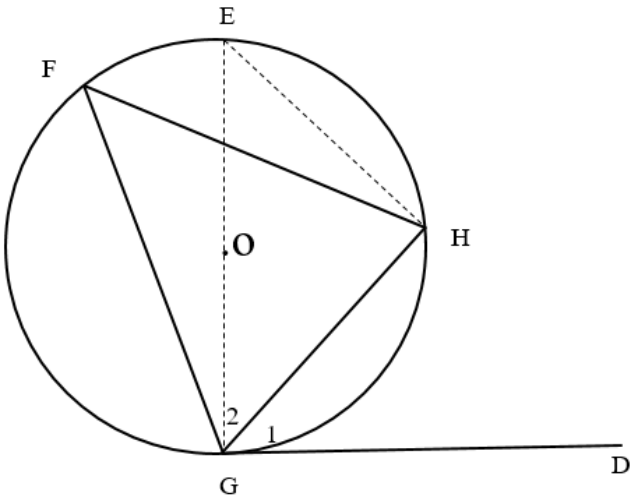
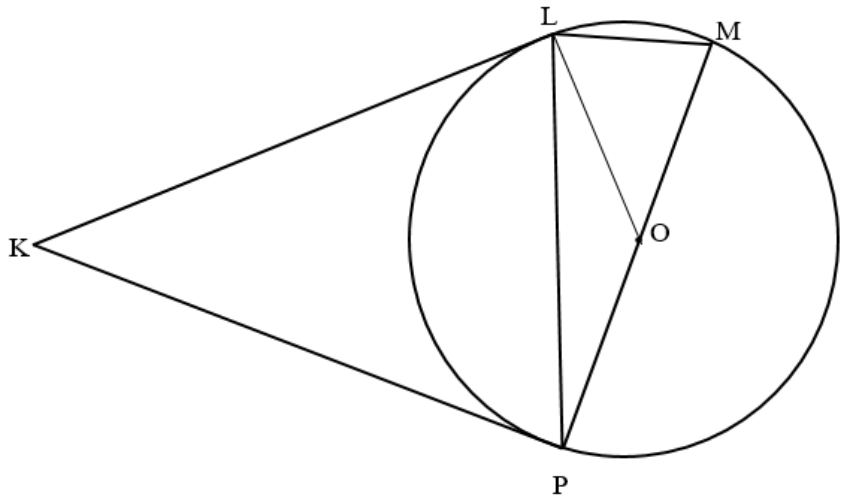
QUESTION 10/*VRAAG 10*

10.1	 Construction: Draw diameter GOE. Join EH *Konstruksie: Trek middellyn GOE. Verbind EH* Proof/*Bewys*: $\hat{G}_1 + \hat{G}_2 = 90°$ (tangent $\perp$ diameter) / (*raaklyn* $\perp$ *radius*) $E\hat{H}G = 90°$ ($\angle$ in the semi circle) / ($\angle$ *in semi - sirkel*) $\hat{G}_2 + \hat{E} = 90°$ (sum of $\angle$s of Δ) / (*som van die* $\angle$*e van 'n* Δ) $\therefore \hat{G}_1 + \hat{G}_2 = \hat{G}_2 + \hat{E}$ $\therefore \hat{G}_1 = \hat{E}$ But/*Maar*: $\hat{E} = \hat{F}$ ($\angle$s in the same segment)/ ($\angle$*e in dieselfde segment*) $\therefore D\hat{G}H = \hat{F}$	✓ construction / *konstruksie* ✓ S/R ✓ S/R ✓ S ✓ S/R (5)

10.2



10.2.1	Kite / *Vlieër*	✓ answer / *antwoord* (1)
10.2.2	$\hat{KLO}=90°$ (tan ⊥ rad.) / (*raaklyn ⊥ radius*) $\hat{KPO}=90°$ (tan ⊥ rad.) / (*raaklyn ⊥ radius*) $\hat{MLP}=90°$ ($\angle$s in the semi circle) / ($\angle e$ *in 'n semi-sirkel*)	✓ S ✓ R ✓ S ✓ S ✓ R (5)
10.2.3	$\hat{KLO}+\hat{KPO}=90°+90°$ $=180°$ $\therefore$ KLOP is a cyclic quad. (Opp. $\angle$s are supp.) KLOP *is 'n koordevierhoek* (*Teenoorst*. $\angle e$ *is* suppl.)	✓ S ✓ R (2)
10.2.4	$\hat{K}+\hat{LOP}=180°$ (Opp. $\angle$s of cyclic quad.) But $\hat{M}=67°$ ($\angle$s opp. equal sides) $\therefore \hat{LOP}=67°+67°$ (Ext. $\angle$ of Δ) $=134°$ $\therefore \hat{K}+134°=180°$ $\therefore \hat{K}=46°$ **OR/*OF*** $\hat{M}=67°$ ($\angle$s opp. = sides) $\hat{LOM}=46°$ ($\angle s$ of Δ) $\therefore \hat{K}=46°$ (ext. $\angle$ of cyclic quad.)	✓ S ✓ R ✓ S/R ✓ value of $\hat{LOP}$ *waarde van* $\hat{LOP}$ ✓ value of $\hat{K}$ / *waarde van* $\hat{K}$ **OR/*OF*** ✓ S ✓ R ✓ value of / *waarde van* $\hat{LOM}$ ✓ value of / *waarde van* $\hat{K}$ ✓ reason / *rede* (5)
		[18]

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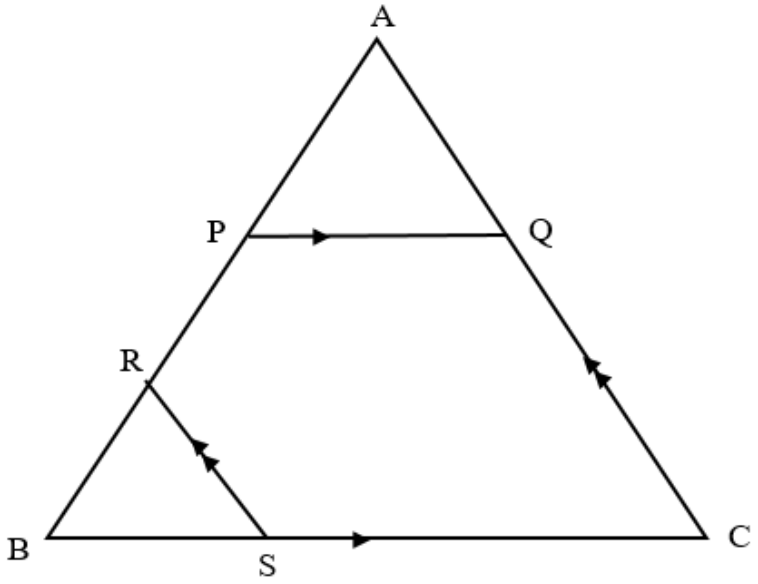
QUESTION 11/*VRAAG 11*



	$\frac{AP}{PB}=\frac{3}{5}$ (Prop. theorem; PQ||BC) (*Verhouding stelling*; PQ||BC) $\frac{AP}{PR+BR}=\frac{3}{5}$ $\therefore 5AP=3PR+3BR$ $\frac{BR}{RA}=\frac{1}{3}$ (Prop. theorem; RS||AC) (*Verhouding stelling*; RS||AC) $\frac{AP}{AP+PR}=\frac{1}{3}$ $3BR=AP+PR$ $\therefore 5AP=3PR+AP+PR$ $4AP=4PR$ $\therefore AP=PR$	✓ S/R ✓ S ✓ simplification / *vereenvoudiging* ✓ S/R ✓ S ✓ simplification / *vereenvoudiging* ✓ substitution / *vervanging* (7)
		[7]

TOTAL/TOTAAL: 150

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