basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL
SENIOR CERTIFICATE/
NASIONALE
SENIOR SERTIFIKAAT

GRADE/*GRAAD* 12

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2019

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/*PUNTE*: 150

These marking guidelines consist of 26pages.
Hierdie nasienriglyne bestaan uit 26 bladsye.

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:

- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Om antwoorde/waardes te aanvaar om 'n probleem op te los, word NIE toegelaat NIE.*

GEOMETRY • *MEETKUNDE*	
S	**A mark for a correct statement** **(A statement mark is independent of a reason)**
	'n Punt vir 'n korrekte bewering ***('n Punt vir 'n bewering is onafhanklik van die rede)***
R	**A mark for the correct reason** **(A reason mark may only be awarded if the statement is correct)**
	'n Punt vir 'n korrekte rede ***('n Punt word slegs vir die rede toegeken as die bewering korrek is)***
S/R	**Award a mark if statement AND reason are both correct**
	Ken 'n punt toe as die bewering EN rede beide korrek is

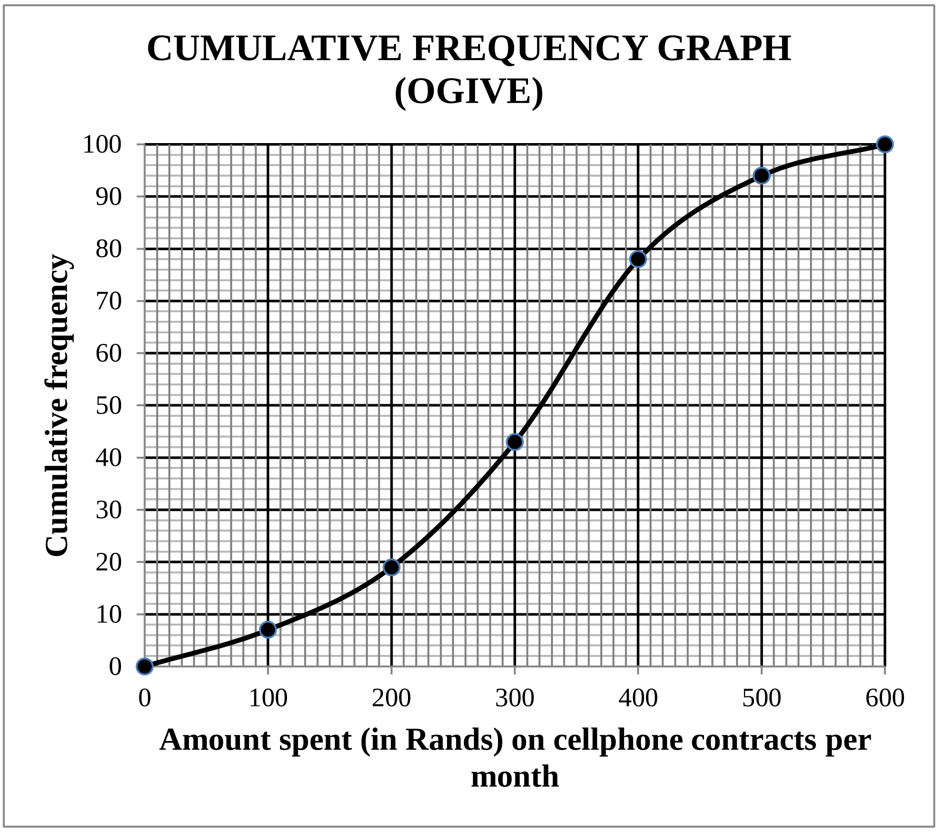
QUESTION/*VRAAG* 1

Monthly income (in rands) **Maandelikse inkomste (in rand)**	9 000	13 500	15 000	16 500	17 000	20 000
Monthly repayment (in rands) **Maandelikse paaiement (in rand)**	2 000	3 000	3 500	5 200	5 500	6 000

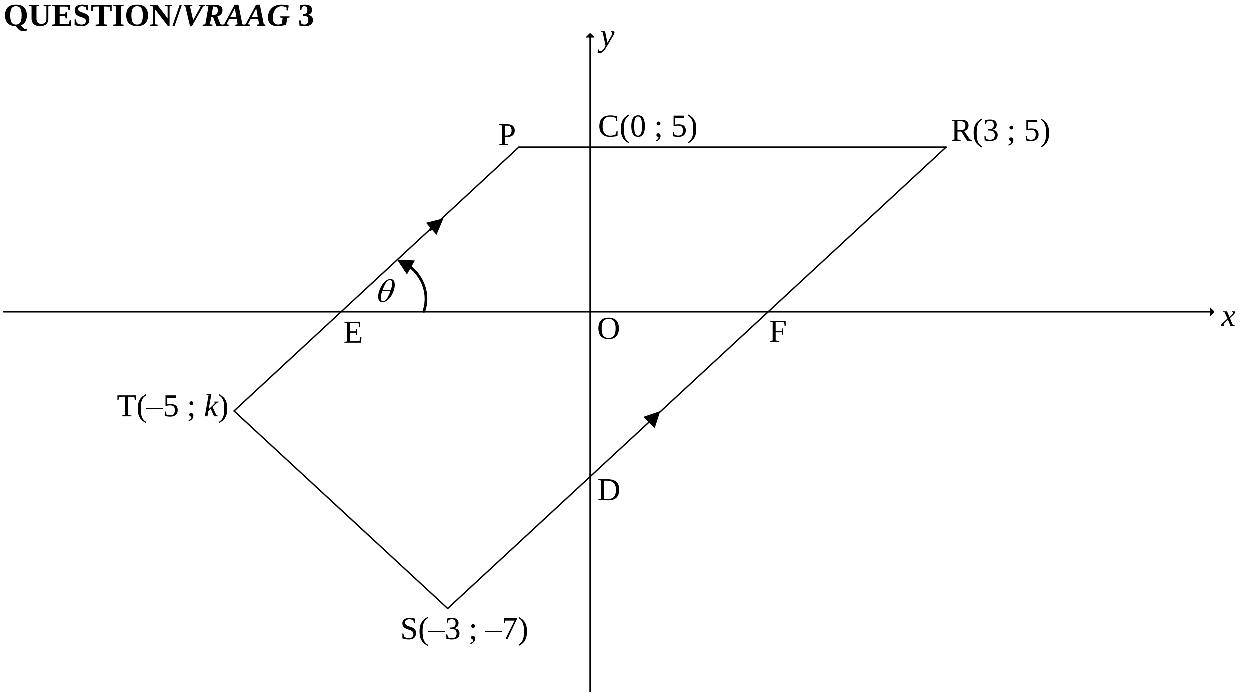
1.1	$a = -1946{,}875... = -1946{,}88$ $b = 0{,}41$ $\hat{y} = -1946{,}88 + 0{,}41x$ Answer only: Full marks	✓ $a = -1946{,}88$ ✓ $b = 0{,}41$ ✓ equation (3)
1.2	Monthly repayment $\approx$ R3 727,16 (calculator) *Maandelikse paaiement* $\approx$ R3 727,16 **OR** $\hat{y} = -1946{,}88 + 0{,}41(14000)$ $\approx$ R3 793,12	✓✓ answer (2) ✓ substitution ✓ answer (2)
1.3	$r = 0{,}946 \ldots \approx 0{,}95$	✓ answer (1)
1.4	Not to spend R9 000 per month because the point (18 000 ; 9 000) lies very far from the least squares regression line. **OR** D *Spandeer nie R9 000 per maand nie, want die punt (18 000 ;9 000) lê baie ver van die kleinste-kwadrate regressielyn.* ***OF*** *D*	✓✓ answer (2)
		[8]

QUESTION/*VRAAG* 2

2.1	Number people paid R200 or less = 19 *Aantal mense wat R200 of minder betaal het = 19*	✓ answer (1)
2.2	$7+12+a+35+b+6=100$ $a=40-b$ $309=\frac{(50\times7)+(150\times12)+(250\times a)+(350\times35)+(450\times b)+(550\times6)}{100}$ $309=\frac{(50\times7)+(150\times12)+(250\times(40-b))+(350\times35)+(450\times b)+(550\times6)}{100}$ 350 + 1800 + 10000 – 250b + 12250 + 450b +3300 = 30900 200b = 3200 b = 16 a = 24 **OR/*OF*** $7+12+a+35+b+6=100$ $b=40-a$ $309=\frac{(50\times7)+(150\times12)+(250\times a)+(350\times35)+(450\times b)+(550\times6)}{100}$ $309=\frac{(50\times7)+(150\times12)+(250\times a)+(350\times35)+(450\times(40-a))+(550\times6)}{100}$ 350 + 1800 + 250a + 12250 + 1800 – 450a = 30900 200a = 4 800 a = 24 b = 16	✓ $\sum x=100$ ✓ $a=40-b$ ✓ $\sum fX$ ✓ $\sum\frac{fX}{n}=309$ ✓ 200b = 3200 (5) ✓ $\sum x=100$ ✓ $b=40-a$ ✓ $\sum fX$ ✓ $\sum\frac{fX}{n}=309$ ✓ 200a = 4 800 (5)
2.3	Modal class/*modale klas:* $300<x\leq400$	✓ answer (1)

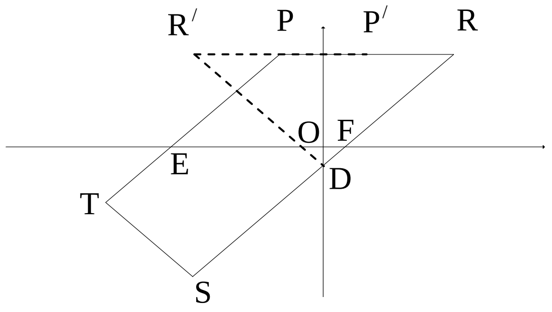
2.4		✓ grounded at (0 ; 0) ✓ (600 ; 100) ✓ cumulative frequencies for *y*-coordinates ✓ smooth shape (4)
2.5	Number of people/*Aantal mense* = 100 – 82 [accept 80 – 84 people] 18 people paid more than R420 per month/. [accept 16 – 20 people] *18 mense betaal meer as R420 per maand* Answer only: Full marks	✓ 82 ✓ answer (2)
		[13]

QUESTION/*VRAAG* 3

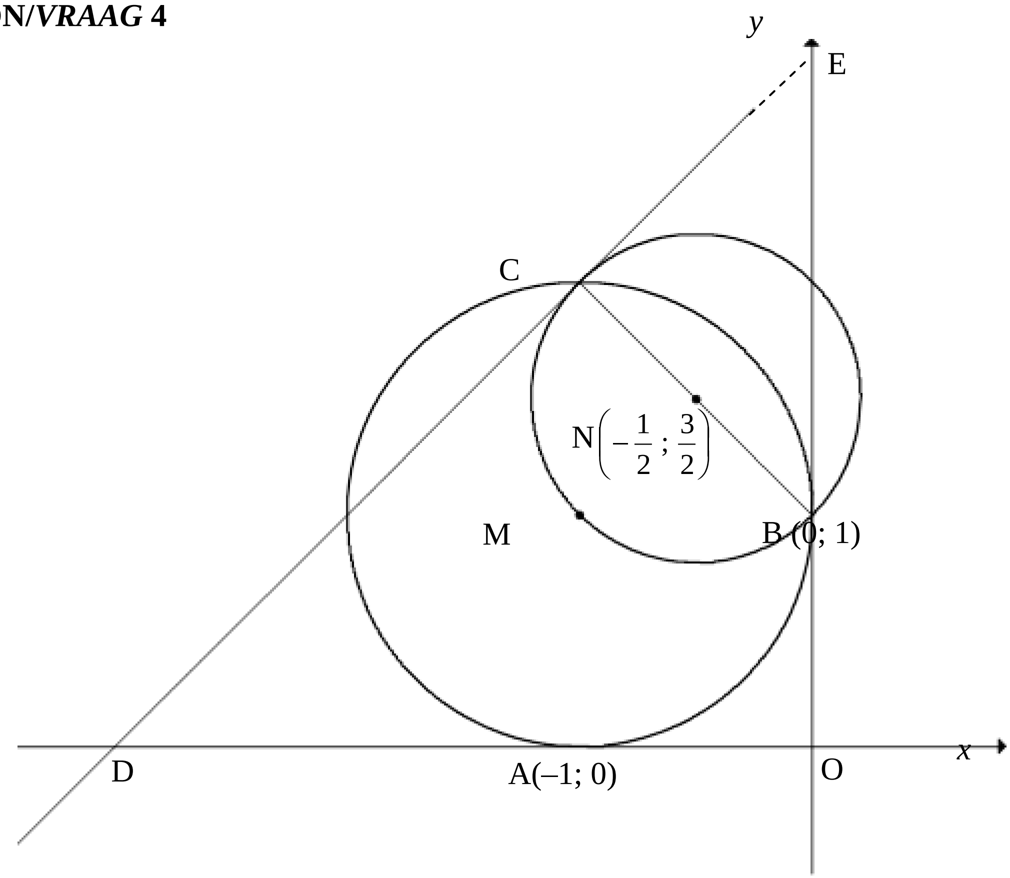


3.1	Equation of PR: $y = 5$	✓ answer (1)
3.2.1	$m_{RS} = \dfrac{y_2 - y_1}{x_2 - x_1}$ $m_{RS} = \dfrac{5-(-7)}{3-(-3)} = \dfrac{12}{6}$ $= 2$ Answer only: Full marks	✓ substitution of R & S into gradient formula ✓ answer (2)
3.2.2	$m_{RS} = m_{PT}$ [PT ∥ RS] $\tan\theta = 2$ $\theta = 63{,}43°$	✓ $m_{RS} = m_{PT}$ ✓ $\tan\theta = 2$ ✓ $\theta = 63{,}43°$ (3)
3.2.3	Equation of RS: $y - 5 = 2(x-3)$ or $y-(-7) = 2(x-(-3))$ or $5 = 2(3)+c$ $y - 5 = 2x - 6$ $y + 7 = 2x + 6$ $c = -1$ $y = 2x - 1$ $y = 2x - 1$ $y = 2x - 1$ $\therefore$ D(0 ; –1) **OR/OF** $m_{RS} = m_{RD} = m_{DS}$ $2 = \dfrac{5-y}{3-0} = \dfrac{y+7}{0-(-3)}$ $\therefore y = -1$ $\therefore$ D(0 ; –1) Answer only: Full marks	✓ substitution ✓ equation of RS ✓ coordinates of D (3) ✓ equating gradients ✓ value of y ✓ coordinates of D (3)

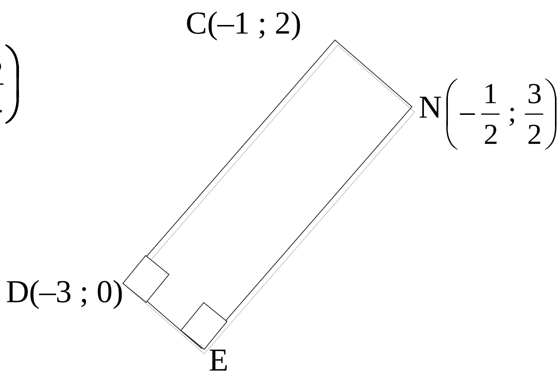
3.3	$\text{ST} = 2\sqrt{5} = \sqrt{[-5-(-3)]^2 + (k-(-7))^2}$ $20 = 4 + (k+7)^2$ $(k+7)^2 = 16$ $k + 7 = \pm 4$ $k = -11$ or $k = -3$ $\therefore k = -3$ **OR** $\text{ST} = 2\sqrt{5} = \sqrt{[-5-(-3)]^2 + (k-(-7))^2}$ $20 = 4 + k^2 + 14k + 49$ $k^2 + 14k + 33 = 0$ $(k+11)(k+3) = 0$ $k = -11$ or $k = -3$ $\therefore k = -3$	✓ substitute S and T into distance formula ✓ isolate square ✓ square root both sides ✓ answer (4) ✓ substitute S and T into distance formula ✓ standard form ✓ factors ✓ answer (4)
3.4	Method: translation T→S: $(x\,;y) \to (x+2\,;y-4)$ $\therefore$ by symmetry: D→N: D(0 ; –1) → N(0 + 2 ; –1 – 4) $\therefore$ N(2 ; –5) Answer only: Full marks **OR** Midpoint of TN = Midpoint of SD $\frac{x+(-5)}{2} = \frac{-3+0}{2}$ and $\frac{y+(-3)}{2} = -\frac{-7+(-1)}{2}$ $x = 2$ and $y = -5$ $\therefore$ N(2 ; – 5) Answer only: Full marks	✓ method ✓ *x*-coordinate ✓ *y*-coordinate (3) ✓ method: midpoint of diagonals ✓ *x*-coordinate ✓ *y*-coordinate (3)

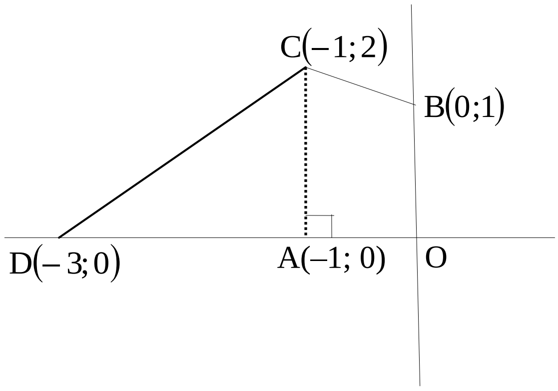
3.5		
	β is the inclination of RS $\therefore\ \beta = 63{,}434...°$ $\hat{OFD} = 63{,}434...°$ [vert opp $\angle$s] $\hat{ODF} = 90° - 63{,}434...° = 26{,}565...°$ $R\hat{D}R^{/} = 2(26{,}565...°) = 53{,}13°$	✓ $\beta = 63{,}43°$ ✓ $\hat{ODF} = 26{,}57°$ ✓ answer (3)
	OR PEFR is a ||m [both pairs of opp sides ||] $\therefore \hat{R} = \theta = 63{,}434...°$ [opp $\angle$s of ||m] $R\hat{R}^{/}D = 63{,}434...°$ [$\angle$s opp = sides:RD = R$^{/}$D] $R\hat{D}R^{/} = 180° - (63{,}43° + 63{,}43°)$ [sum of $\angle$s in Δ] $R\hat{D}R^{/} = 53{,}13°$	✓ $\hat{R} = 63{,}43°$ ✓ $R\hat{R}^{/}D = 63{,}43°$ ✓ answer (3)
	OR $\tan O\hat{D}F = \frac{3}{6}$ $O\hat{D}F = 26{,}565..°$ $R\hat{D}R^{/} = 2(26{,}565...°) = 53{,}13°$	✓ trig ratio ✓ $O\hat{D}F = 26{,}565..°$ ✓ answer (3)
	OR $R^{/}(-3\,;5)$ [reflection of R(3 ; 5) about the *y*-axis] $RD = \sqrt{(3-0)^2 + (5-(-1)^2}$ $RD = \sqrt{45} = R^{/}D$ or $3\sqrt{5}$ or $6{,}71$ $(RR^{/})^2 = (\sqrt{45})^2 + (\sqrt{45})^2 - 2(\sqrt{45})(\sqrt{45})(\cos R\hat{D}R^{/})$ $6^2 = 45 + 45 - 2(45)(\cos R\hat{D}R^{/})$ $\cos R\hat{D}R^{/} = \frac{45+45-36}{2(45)}$ $\cos R\hat{D}R^{/} = \frac{3}{5}$ $\therefore\ R\hat{D}R^{/} = 53{,}13°$	✓ $R^{/}(-3\,;5)$ **OR** $RD = \sqrt{45} = R^{/}D$ ✓ substitution into cosine rule ✓ answer (3)
		[19]

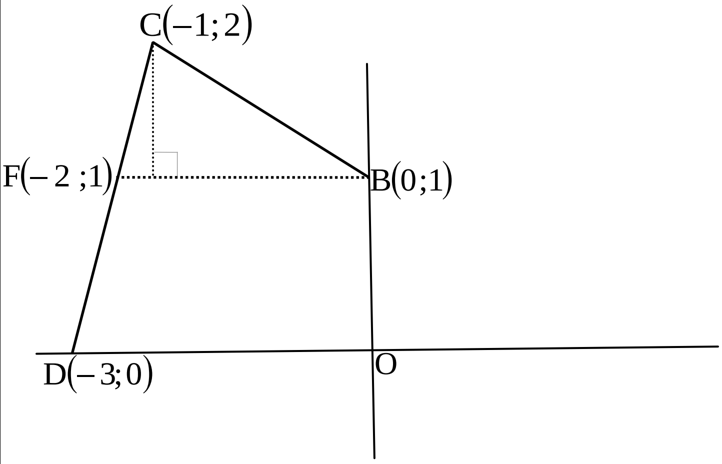
QUESTION/*VRAAG* 4

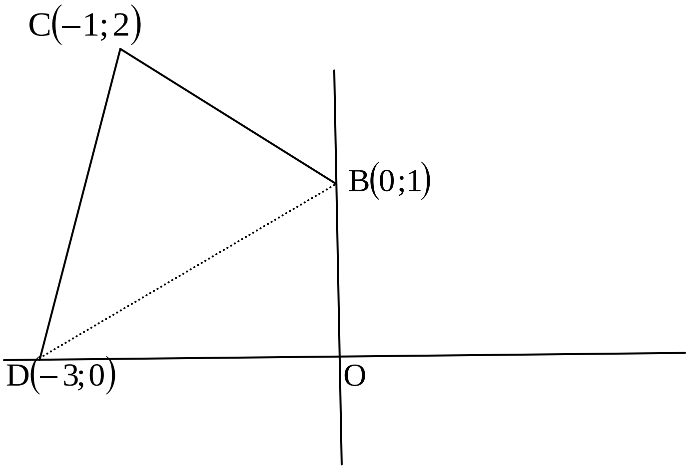


4.1	M(–1;1) $(x+1)^2+(y-1)^2=1$ $\boxed{\text{Answer only: Full marks}}$	✓ M(–1;1) ✓LHS ✓ RHS (3)
4.2	Midpoint of CB, N: (– 0,5 ; 1,5) $\therefore \frac{x_C+0}{2}=-\frac{1}{2}$ and $\frac{y_C+1}{2}=\frac{3}{2}$ $\boxed{\text{Answer only: Full marks}}$ $\therefore$ C(– 1 ; 2) **OR** B→N: $(x;y)\rightarrow(x-0{,}5;y+0{,}5)$ N→C: $\boxed{\text{Answer only: Full marks}}$ $(x;y)\rightarrow(x-0{,}5;y+0{,}5)$ $\therefore$ C(– 0,5 – 0,5 ; 1,5 + 0,5) $\therefore$ C(– 1 ; 2)	✓*x* value ✓ *y* value (2) ✓*x* value ✓ *y* value (2)

4.3	$m_{\text{radius}} = \dfrac{2-1}{-1-0}$ OR $\dfrac{2-(-\frac{1}{2})}{-1-\frac{3}{2}}$ OR $\dfrac{0-(-\frac{1}{2})}{1-\frac{3}{2}}$ $= -1$ $\therefore m_{\text{tangent}} = 1$ $y = mx + c$ $y = x + c$ $2 = 1(-1) + c$ $c = 3$ $\therefore y = x + 3$ $y - x = 3$ **OR** $m_{\text{radius}} = \dfrac{2-1}{-1-0}$ $= -1$ $\therefore m_{\text{tangent}} = 1$ $y - y_1 = m(x - x_1)$ $y - y_1 = 1(x - x_1)$ $y - 2 = 1(x - (-1))$ $y - 2 = x + 1$ $\therefore y = x + 3$ $y - x = 3$	✓ m_{radius} ✓ m_{tangent} ✓ substitute (– 1 ; 2) and *m* ✓ simplification (4) ✓ m_{radius} ✓ m_{tangent} ✓ substitute (– 1 ; 2) and *m* ✓ simplification (4)
4.4	Tangents to circle: $y = x + 3$ and $y = x + 1$ $\therefore t > 3$ or $t < 1$ Answers only: Full marks	✓ $y = x + 1$ ✓ $t > 3$ ✓ $t < 1$ (3)
4.5	Draw rectangle CNED: Midpt of DN $\left(-\frac{7}{4} ; \frac{3}{4}\right)$ $\therefore E(-\frac{5}{2} ; -\frac{1}{2})$  **OR/*OF*** D(– 3 ; 0) C→N: $(x ; y) \to (x + 0{,}5 ; y - 0{,}5)$ D→E: $D(x ; y) \to E(x + 0{,}5 ; y - 0{,}5)$ $\therefore$ E(– 3 + 0,5 ; 0 – 0,5) $\therefore$ E(– 2,5 ; – 0,5) Answer only: Full marks	✓ midpt of DN ✓*x* value ✓ *y* value (3) ✓ coordinates of D ✓*x* value ✓ *y* value (3)

4.6	 area of trapezium AOBC = $\frac{1}{2}(1+2)(1)$ $= 1\frac{1}{2}$ square units area of ΔACD = $\frac{1}{2}(2)(2)$ = 2 square units area of quadrilateral OBCD = $3\frac{1}{2}$ square units $\therefore 2a^2 = \frac{7}{2}$ $a^2 = \frac{7}{4}$ $a = \frac{\sqrt{7}}{2}$ **OR**	✓ substitution into area of trapezium form ✓ area of trapezium ✓ area of triangle ✓ area of OBCD ✓ equating area OBCD to $2a^2$ (5)

	 BM produced cuts the tangent at F. area of ΔCFB = $\frac{1}{2}(2)(1)$ = 1 square unit area of trapezium BFDO = $\frac{1}{2}(2+3)(1)$ = $2\frac{1}{2}$ square units area of quadrilateral OBCD = $3\frac{1}{2}$ square units $\therefore 2a^2 = \frac{7}{2}$ $a^2 = \frac{7}{4}$ $a = \frac{\sqrt{7}}{2}$ **OR**	✓ area of triangle ✓ substitution into area of trapezium ✓ area of trapezium ✓ area of OBCD ✓ equating area OBCD to $2a^2$ (5)



Join DB

area of $\Delta ODB = \frac{1}{2}(3)(1)$

$= \frac{3}{2}$ square unit

area of $\Delta DCB = \frac{1}{2}\left(2\sqrt{2}\right)\left(\sqrt{2}\right)$

$= 2$ square unit

$\therefore$ area of OBCD = $\frac{3}{2} + 2 =$ square units

$$2a^2 = \frac{7}{2}$$

$$a^2 = \frac{7}{4}$$

$$a = \frac{\sqrt{7}}{2}$$

OR

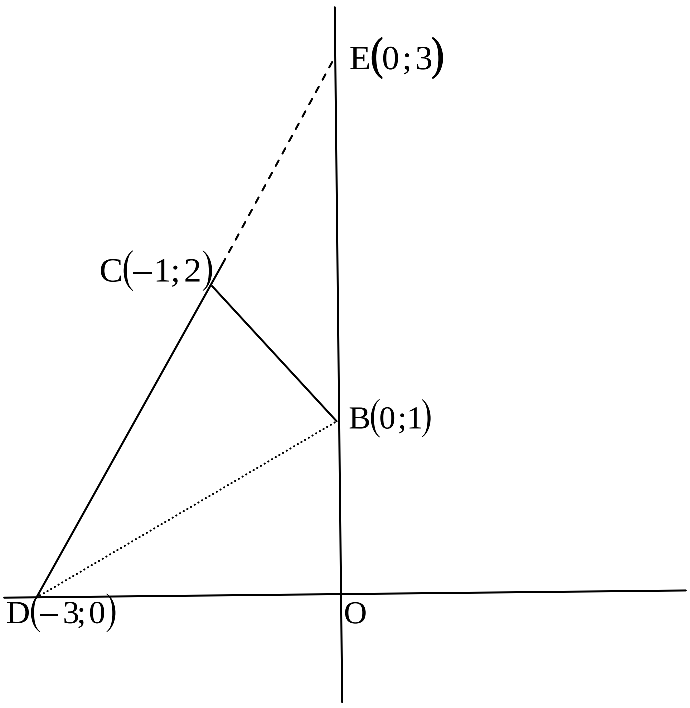
✓ area of Δ

✓ subst into area of Δ

✓ area of Δ
✓ area of OBCD

✓ equating area OBCD to $2a^2$

(5)

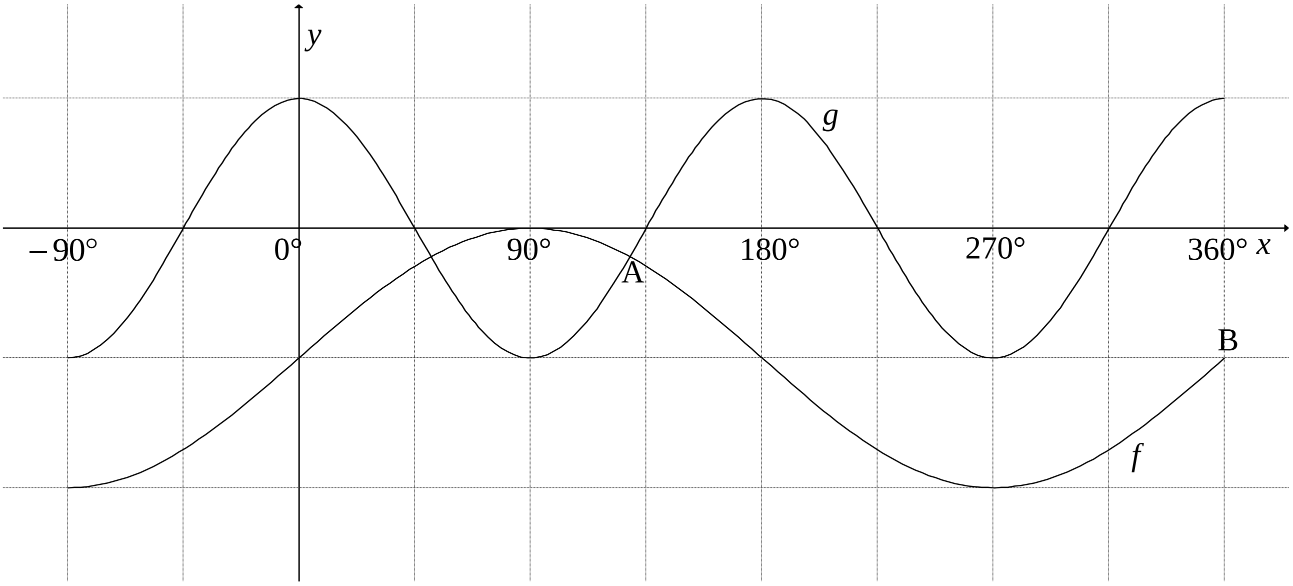
		
	Let E be the point of intersection of DC with the positive *y*–axis. area of ΔDEO = $\frac{1}{2}(3)(3)$ = $\frac{9}{2}$ square unit area of ΔECB = $\frac{1}{2}(2)(1)$ or $\frac{1}{2}(\sqrt{2})(\sqrt{2})$ = 1 square unit area of quadrilateral OBCD = $\frac{9}{2}-1=3\frac{1}{2}$ square units $\therefore 2a^2=\frac{7}{2}$ $a^2=\frac{7}{4}$ $a=\frac{\sqrt{7}}{2}$	✓ area of Δ ✓ subst into area of Δ ✓ area of Δ ✓ area of OBCD ✓ equating area OBCD to $2a^2$ (5)
		[20]

QUESTION/*VRAAG* 5

5.1	$\frac{\sin x}{\cos x.\tan x}+\sin(180°+x)\cos(90°-x)$ $=\frac{\sin x}{\cos x.\frac{\sin x}{\cos x}}+(-\sin x)\sin x$ $=1-\sin^2 x$ $=\cos^2 x$	✓ $-\sin x$ ✓ $\sin x$ ✓ $\tan x=\frac{\sin x}{\cos x}$ ✓ $1-\sin^2 x$ ✓ $\cos^2 x$ (5)
5.2	$\frac{\sin^2 35°-\cos^2 35°}{4\sin 10°\cos 10°}$ $=\frac{-(\cos^2 35°-\sin^2 35°)}{2(2\sin 10°\cos 10°)}$ $=\frac{-\cos 70°}{2\sin 20°}$ $=\frac{-\cos 70°}{2\cos 70°}$ **OR** $=\frac{-\sin 20°}{2\sin 20°}$ $=-\frac{1}{2}$	✓ $-(\cos^2 35°-\sin^2 35°)$ ✓ $-\cos 70°$ ✓ $2\sin 20°$ ✓ answer (4)
5.3	$2\sin^2 77°=2[\sin(90°-13°)]^2$ $=2\cos^2 13°$ $=2\cos^2 13°-1+1$ $=\cos 26°+1$ $=m+1$ **OR** $1-2\sin^2 77°=\cos 154°$ $2\sin^2 77°=1-\cos 154°$ $=1-(-\cos 26°)$ $=1+m$	✓ using co-ratio ✓ reduction ✓ $2\cos^2 13°-1=\cos 26°$ ✓ answer (4) ✓ $1-2\sin^2 77°=\cos 154°$ ✓ $2\sin^2 77°=1-\cos 154°$ ✓ reduction ✓ answer (4)
5.4.1	$\sin(x+25°)\cos 15°-\cos(x+25°)\sin 15°=\tan 165°$ $\sin(x+25°-15°)=-0{,}2679...$ *OR* $-2+\sqrt{3}$ $\sin(x+10°)=-0{,}2679...$ *OR* $-2+\sqrt{3}$ $x+10°=195{,}54°+k.360°$ or $x+10°=344{,}46°+k.360°$ $x=185{,}54°+k.360°;\ k\in Z$ or $x=334{,}46°+k.360°;k\in Z$ **OR/*OF***	✓✓ $\sin(x+10°)$ ✓ $-0{,}2679...$ ✓ $195{,}54°$ & $344{,}46°$ ✓ $185{,}54°$ & $334{,}46°$ ✓ $+k.360°;\ k\in Z$ (6)

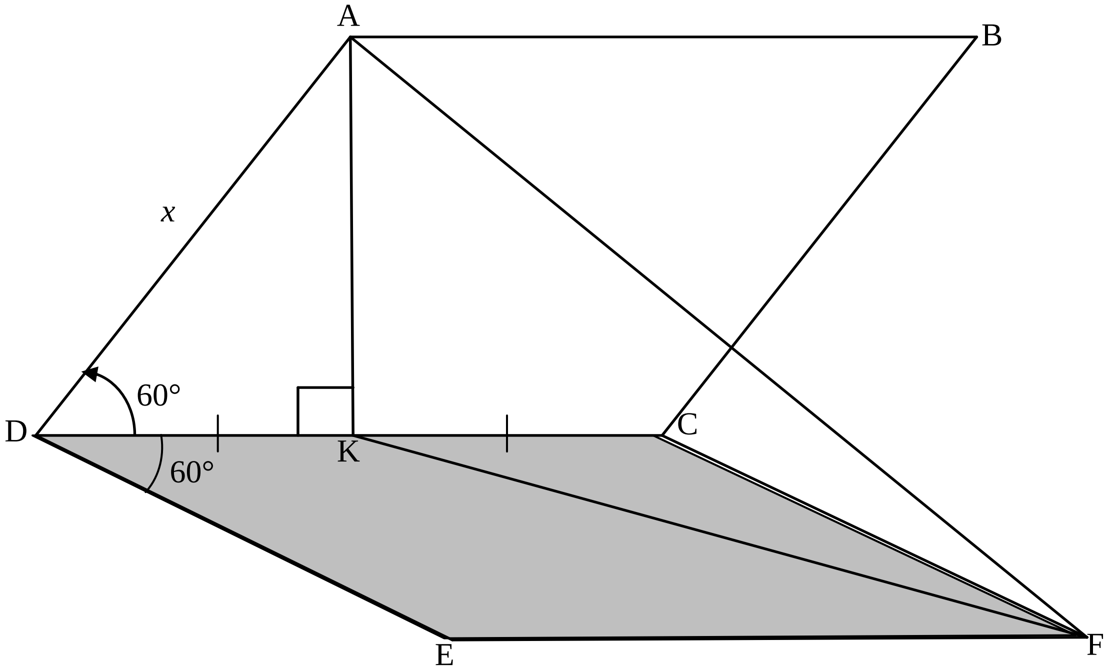
	$\sin(x+25°)\sin 75° - \cos(x+25°)\cos 75° = \tan 165°$ $-(\cos(x+25°)\cos 75° - \sin(x+25°)\sin 75°) = -0,2679...$ $\cos(x+100°) = 0,2679...$ ref. $\angle = 74.4577\ldots°$ $x+100° = 74,46° + k.360°$ *or* $x+100° = 285,54° + k.360°$ $x = -25,54° + k.360°;\ k \in Z$ or $x = 185,54° + k.360°;\ k \in Z$	✓✓ $\cos(x+100°)$ ✓ $-0,2679...$ ✓ 74,46° & 285,54° ✓ $-25,54°$ & 185,54° ✓ $+k.360°;\ k \in Z$ (6)
5.4.2	$f(x) = \sin(x+10°)$ **Answers only: Full marks** For minimum value of $\sin x$: $x = 270°$ For minimum value of $\sin(x+10°)$: $x = 260°$	✓ $f(x) = \sin(x+10°)$ ✓ 270° ✓ answer (3)
		[22]

QUESTION/*VRAAG* 6



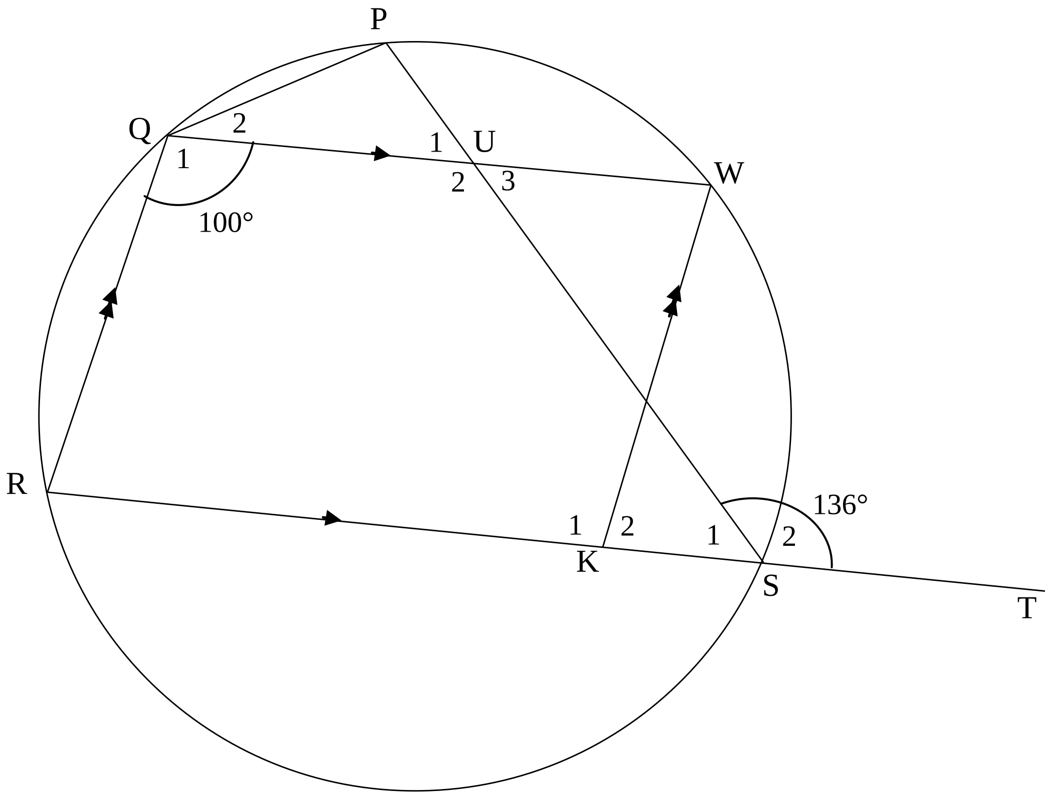
6.1	Range of f: $y \in [-2\ ;0]$ **OR** $-2 \leq y \leq 0$	✓ critical values ✓ notation (2)
6.2	$x \in (90°\ ;\ 270°)$ OR $x \in [90°\ ;\ 270°]$	✓ critical values ✓ notation (2)
6.3	$\text{PQ} = \cos 2x - (\sin x - 1)$ $= 1 - 2\sin^2 x - \sin x + 1$ $= -2\sin^2 x - \sin x + 2$ $\sin x = -\frac{b}{2a}$ $= \frac{-(-1)}{2(-2)}$ $\sin x = -\frac{1}{4}$ $\therefore\ x = 194{,}48°$ or $x = 345{,}52°$	✓ $\text{PQ} = \cos 2x - (\sin x - 1)$ ✓ $\cos 2x = 1 - 2\sin^2 x$ ✓substitution into formula ✓ $\sin x = -\frac{1}{4}$ ✓ 194,48° ✓ 345,52° (6)
		[10]

QUESTION/*VRAAG* 7



7.1	$\sin 60° = \frac{AK}{x}$ $AK = x \sin 60°$ or $\frac{\sqrt{3}}{2}x$ or $0{,}866x$	✓ trig ratio ✓ answer (2)
7.2	$K\hat{C}F = 120°$	✓ answer (1)
7.3	$KF^2 = CF^2 + CK^2 - 2CF.CK\cos K\hat{C}F$ $= x^2 + \left(\frac{x}{2}\right)^2 - 2x\left(\frac{x}{2}\right)\cos 120°$ $= x^2 + \frac{x^2}{4} - x^2\left(-\frac{1}{2}\right)$ $= \frac{7x^2}{4}$ $KF = \frac{\sqrt{7}x}{2}$ $A\hat{K}F = y$ Area $\Delta AKF = \frac{1}{2}.AK.KF\sin A\hat{K}F$ $= \frac{1}{2}.\frac{\sqrt{3}x}{2}.\frac{\sqrt{7}x}{2}\sin y$ $= \frac{x^2\sqrt{21}\sin y}{8}$	✓ correct use of cosine rule ✓substitution ✓ $\cos 120° = -\frac{1}{2}$ ✓ $KF = \frac{\sqrt{7}x}{2}$ ✓ correct use of area rule ✓substitution ✓ answer in terms of *x* and *y* (7)
		[10]

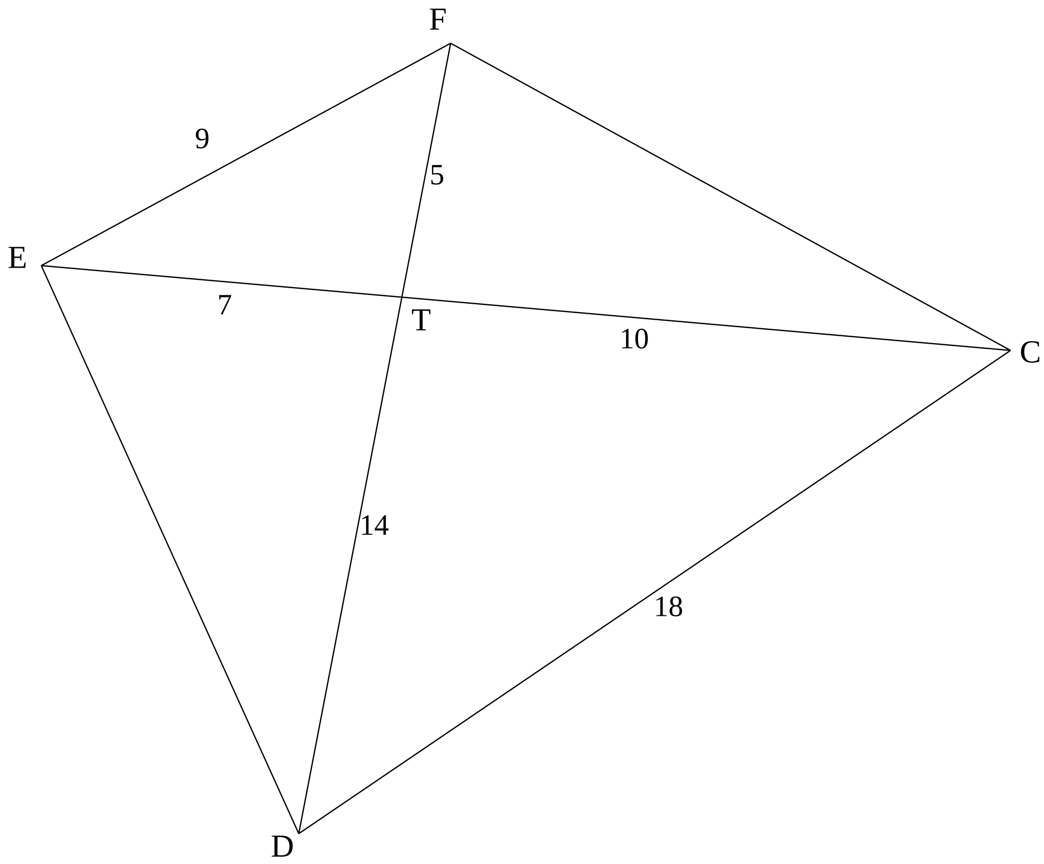
QUESTION/*VRAAG* 8



8.1.1	$\hat{R} = 80°$ [co-int ∠s/*ko-binne* ∠*e*; QW ‖ RK]	✓S ✓R (2)
8.1.2	$\hat{P} = 100°$ [opp ∠s of cyclic quad/*teenoorst* ∠*e v koordevh*]	✓S ✓R (2)
8.1.3	$P\hat{Q}R = 136°$ [ext ∠ of cyclic quad/*buite* ∠ *v koordevh*] $\hat{Q}_2 = 36°$ **OR** $P\hat{U}W = \hat{S}_2 = 136°$ [corresp ∠s/*ooreenkomstige* ∠*e*; QW ‖ RK] $P\hat{Q}W + \hat{P} = P\hat{U}W$ [ext ∠s of/*buite* ∠ *van* Δ QPU] $P\hat{Q}W + 100° = 136°$ $P\hat{Q}W = 36°$ **OR** $\hat{U}_3 = 180° - 136° = 44°$ [co-int ∠s/*ko-binne* ∠*e*; QW ‖ RK] $\hat{U}_1 = \hat{U}_3 = 44°$ [vert opp ∠s/*regoorstaande* ∠*e*] $P\hat{Q}W = 180° - (100 + 44°)$ [sum of ∠s in Δ/*som* ∠*e van* Δ] $P\hat{Q}W = 36°$	✓S ✓R ✓S (3) ✓S ✓R ✓S (3) ✓S ✓R ✓S (3)

8.1.4	$\hat{U}_2 = \hat{S}_2 = 136°$ [alt $\angle$s/*verwiss* $\angle e$; QW $\parallel$ RK]	✓S ✓R (2)
	OR	
	$\hat{U}_2 = 100° + 36°$ $= 136°$ [ext $\angle$s of/*buite* $\angle$ *van* ΔQPU]	✓S ✓R (2)
	OR	
	$\hat{U}_2 = P\hat{U}W = 136°$ [vert opp $\angle$s/*regoorstaande* $\angle e$]	✓S ✓R (2)
	OR	
	$\hat{U}_2 = 180° - \hat{U}_3$ $= 180° - 44°$ $= 136°$ [$\angle$s on a str line/$\angle e$ *op reguitlyn*]	✓S ✓R (2)

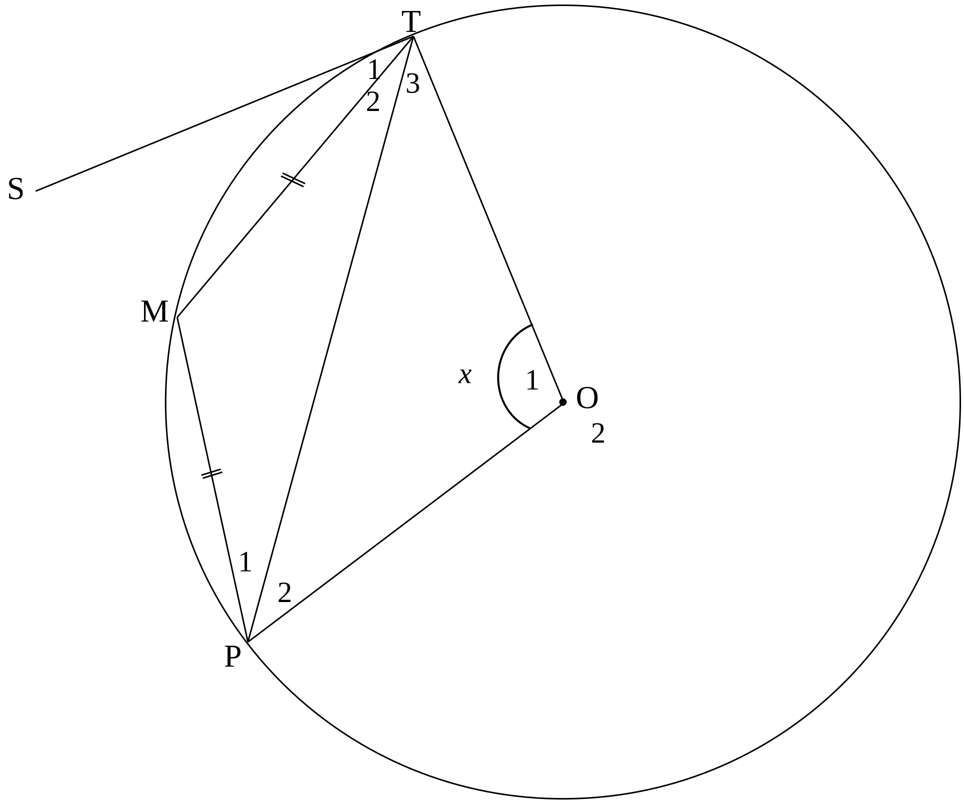
8.2



8.2.1	In ΔEFT and ΔDCT: $\frac{EF}{CD}=\frac{9}{18}=\frac{1}{2}$ $\frac{FT}{TC}=\frac{5}{10}=\frac{1}{2}$ $\frac{ET}{TD}=\frac{7}{14}=\frac{1}{2}$ $\therefore \Delta EFT \,\vert\vert\vert\, \Delta DCT$ [Sides of Δ in prop/ *sye van Δ in dieselfde verh*] $\therefore E\hat{F}D = E\hat{C}D$ **OR** In ΔFET: $49=25+81-2(5)(9)\cos\hat{F}$ $\cos\hat{F}=\frac{19}{30}$ $\hat{F}=50,7°$ In ΔTDC: $196=100+256-2(10)(18)\cos\hat{C}$ $\cos\hat{C}=\frac{19}{30}$ $\hat{C}=50,7°$	✓✓ all 3 ratios = $\frac{1}{2}$ ✓ $\Delta EFT \,\vert\vert\vert\, \Delta DCT$ ✓ R (4) ✓✓ $\hat{F}=50,7°$ ✓✓ $\hat{C}=50,7°$ (4)

8.2.2	$E\hat{F}D = E\hat{C}D$ [proved in 8.2.1] E, F, C and D are concyclic EFCD is a cyclic quad [converse ∠s in the same segment/ *omgekeerde ∠e in dies segment*] $\therefore D\hat{F}C = D\hat{E}C$ [∠s in the same segment/∠*e in dies segment*]	✓S ✓R ✓ R (3)
		[16]

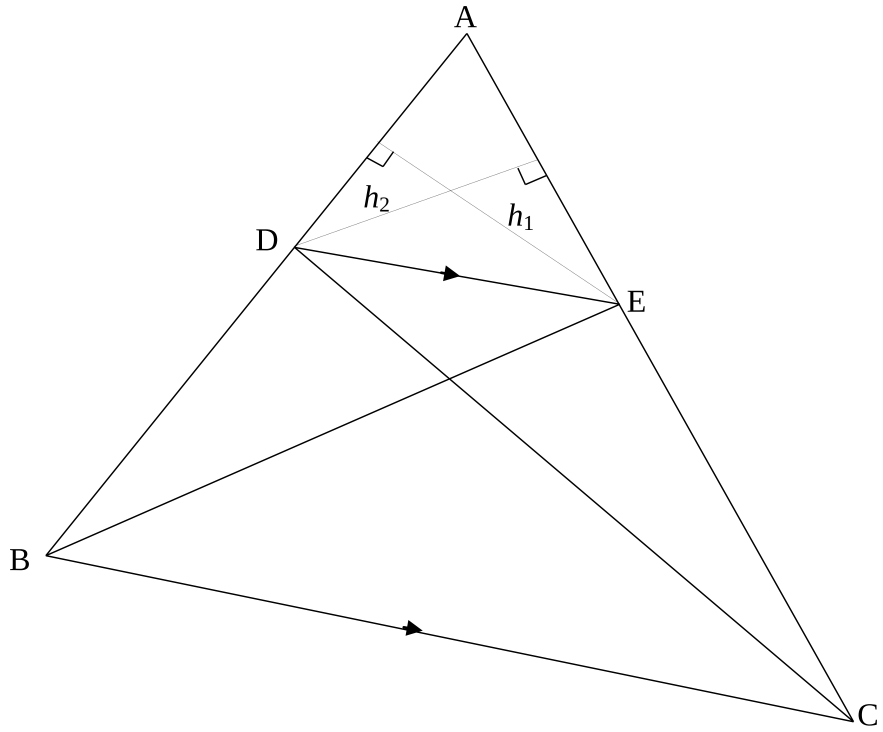
QUESTION/*VRAAG* 9



	$\hat{O}_2 = 360° - x$ [∠s round a pt/*∠e om ´n punt*]	✓ $\hat{O}_2 = 360° - x$
	$\therefore \hat{M} = 180° - \frac{1}{2}x$ [∠ at centre = 2 ×∠ at circumference/ *middelpunts∠ = 2 × omtreks∠*]	✓ $\hat{M} = 180° - \frac{1}{2}x$ ✓R
	$\therefore \hat{T}_2 + \hat{P}_1 = \frac{1}{2}x$ [sum of ∠s in Δ/*som ∠e van Δ*]	✓ $\hat{T}_2 + \hat{P}_1 = \frac{1}{2}x$
	$\therefore \hat{T}_2 = \hat{P}_1 = \frac{1}{4}x$ [∠s opp equal sides/*∠e teenoor gelyke sye*]	✓ $\hat{P}_1 = \frac{1}{4}x$ ✓R
	$\therefore S\hat{T}M = \hat{P}_1 = \frac{1}{4}x$ [tan chord theorem/*raaklyn koordstelling*]	✓R (7)
	OR/*OF*	
	$\hat{O}_2 = 360° - x$ [∠s round a pt/*∠e om ´n punt*]	✓ $\hat{O}_2 = 360° - x$ ✓ S ✓R
	$\therefore \hat{M} = \frac{1}{2}\hat{O}_2$ [∠ at centre = 2 ×∠ at circumference]	✓S
	$\therefore \hat{T}_2 + \hat{P}_1 = 180° - \hat{M}$ [sum of ∠s in Δ/*som ∠e van Δ*]	✓R
	$\therefore \hat{T}_2 = \hat{P}_1$ [∠s opp equal sides/*∠e teenoor gelyke sye*]	
	$= \frac{180° - \hat{M}}{2} = \frac{180° - \frac{1}{2}\hat{O}_2}{2} = \frac{180° - \frac{1}{2}(360° - x)}{2} = \frac{1}{4}x$	✓ S
	$\therefore S\hat{T}M = \frac{1}{4}x$ [tan chord theorem/*raaklyn koordstelling*]	✓R (7)
		[7]

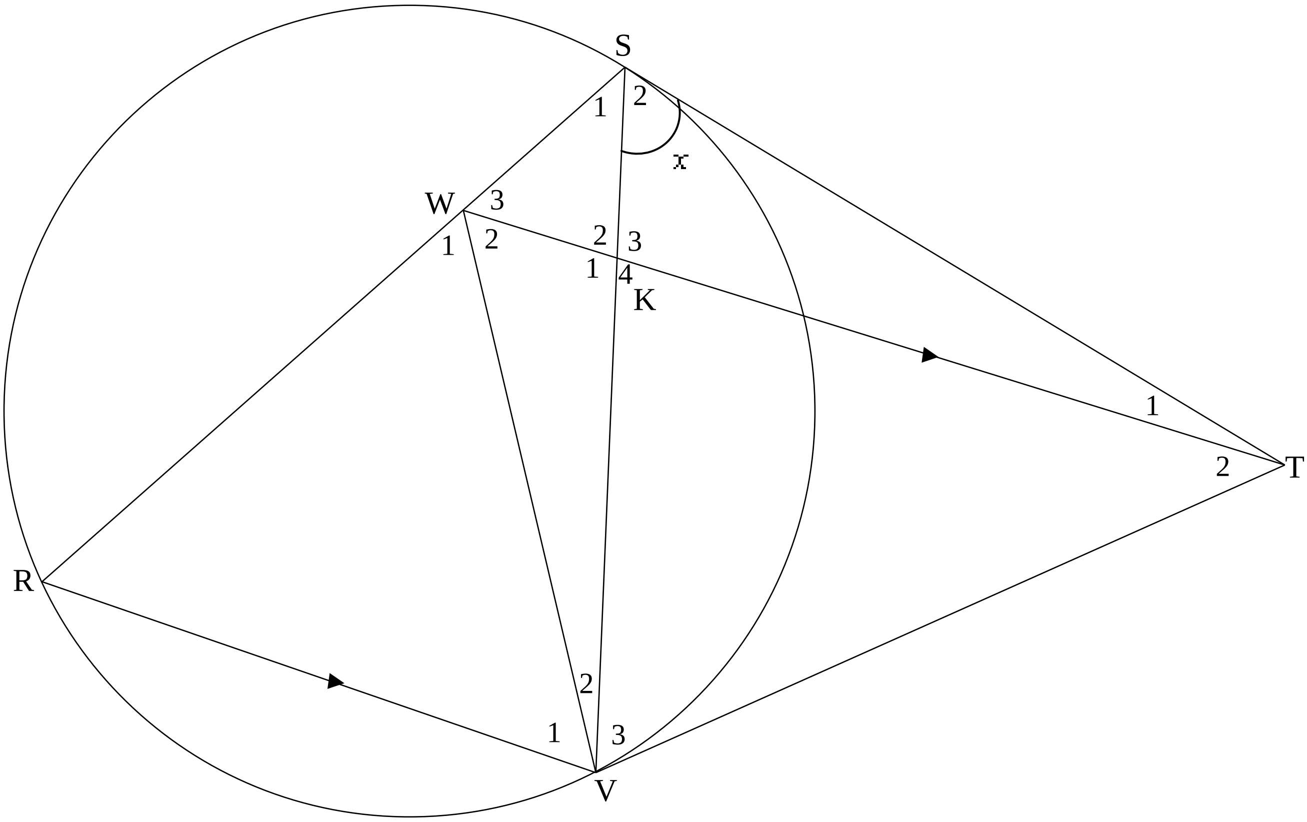
QUESTION/*VRAAG* 10

10.1



10.1	Constr: Draw h_1 from E $\perp$ AD and h_2 from D $\perp$ AE *Konstr: Trek* h_1 *vanaf* E $\perp$ AD *en* h_2 *vanaf* D $\perp$ AE Proof/*Bewys*: $\frac{\text{area } \Delta\text{ADE}}{\text{area } \Delta\text{BDE}} = \frac{\frac{1}{2}\text{AD}\times h_1}{\frac{1}{2}\text{DB}\times h_1} = \frac{\text{AD}}{\text{DB}}$ $\frac{\text{area } \Delta\text{ADE}}{\text{area } \Delta\text{DEC}} = \frac{\frac{1}{2}\text{AE}\times h_2}{\frac{1}{2}\text{EC}\times h_2} = \frac{\text{AE}}{\text{EC}}$ But area ΔBDE = area ΔDEC [same base & height or DE || BC/ *dies basis & hoogte*; of DE || BC] $\therefore \frac{\text{area } \Delta\text{ADE}}{\text{area } \Delta\text{BDE}} = \frac{\text{area } \Delta\text{ADE}}{\text{area } \Delta\text{DEC}}$ $\therefore \frac{\text{AD}}{\text{DB}} = \frac{\text{AE}}{\text{EC}}$	✓ constr/*konstr* **OR reason: common vertex or same height** ✓ $\frac{\text{area } \Delta\text{ADE}}{\text{area } \Delta\text{BDE}} = \frac{\frac{1}{2}\text{AD}\times h_1}{\frac{1}{2}\text{DB}\times h_1}$ ✓ $\frac{\text{area } \Delta\text{ADE}}{\text{area } \Delta\text{DEC}} = \frac{\text{AE}}{\text{EC}}$ ✓ S ✓R ✓ S (6)

10.2



10.2.1	$\hat{V}_3 = x$ [Tans from same point/*raaklyne vanaf dieselfde pt*] $\hat{R} = x$ [tan chord theorem/*raaklyn koordstelling*] $\hat{W}_3 = x$ [corresp ∠s/*ooreenkomstige ∠e*; WT ∥ RV]	✓ S ✓ R ✓ S ✓ R ✓ S ✓ R (6)
10.2.2(a)	$\hat{V}_3 = \hat{W}_3 = x$ [proved in 10.2.1] W, S, T and V are concyclic/*is konsiklies* WSTV is a cyclic quad [converse ∠s in the same segment/ *Omgekeerde ∠e in dieselfde segment*]	✓ S ✓ R (2)
10.2.2(b)	$\hat{W}_2 = \hat{S}_2 = x$ [∠s in the same segment/∠e *in dies segment*] $\hat{V}_1 = \hat{W}_2 = x$ [alt ∠s/*verwiss* ∠e ; WT ∥ RV] But $\hat{R} = x$ [proved in 10.2.1] $\therefore \hat{R} = \hat{V}_1 = x$ ∴ WR = WV [sides opp equal ∠s/*sye teenoor gelyke ∠e*] ΔWRV is isosceles/*is gelykbenig* **OR/*OF***	✓ S ✓R ✓ S/ R ✓ S (4)

	$\hat{S}_2 = \hat{W}_2 = x$ [$\angle$s in the same segment] $\hat{W}_2 = \hat{W}_3 = x$ $\hat{W}_2 + \hat{W}_3 = \hat{R} + \hat{V}_1$ [ext $\angle$ of Δ] $\therefore\ \hat{V}_1 = x = \hat{R}$ $\therefore$ WR = WV [sides opp equal $\angle$s/*sye teenoor gelyke* $\angle$*e*] ΔWRV is isosceles/*is gelykbenig*	✓ S ✓R ✓ S/ R ✓ S (4)
10.2.2(c)	In ΔWRV and/*en* ΔTSV $\hat{R} = \hat{S}_2 = x$ [proved **OR** tan chord theorem] $\hat{V}_1 = \hat{V}_3 = x$ [proved] $\therefore \Delta WRV \mid\mid\mid \Delta TSV$ [$\angle, \angle, \angle$] **OR/*OF*** In ΔWRV and/*en* ΔTSV $\hat{R} = \hat{S}_2 = x$ [proved **OR** tan chord theorem] $\hat{V}_1 = \hat{V}_3 = x$ [proved] $\hat{W}_1 = S\hat{T}V = x$ [sum of $\angle$s in Δ/$\angle$*e van* Δ] $\therefore \Delta WRV \mid\mid\mid \Delta TSV$	✓ S ✓ S ✓ R (3) ✓ S ✓ S ✓ S (3)
10.2.2(d)	$\frac{RV}{SV} = \frac{WR}{TS}$ [$\Delta WRV \mid\mid\mid \Delta TSV$] $\therefore WR \times SV = RV \times TS$ $\frac{WR}{SR} = \frac{KV}{SV}$ [prop theorem/*eweredighst*; WT $\parallel$ RV] $\therefore WR \times SV = KV \times SR$ $\therefore RV \times TS = KV \times SR$ $\therefore \frac{RV}{SR} = \frac{KV}{TS}$ **OR/*OF*** In ΔRVS and/*en* ΔVKT $S\hat{V}R = \hat{K}_4$ [alt $\angle$s, WT $\parallel$ RV] $S\hat{R}V = \hat{V}_3$ [proven] $\Delta RVS \mid\mid\mid \Delta VKT$ [$\angle, \angle, \angle$] $\therefore \frac{RV}{SR} = \frac{KV}{VT}$ but VT = ST [tans from same point] $\therefore \frac{RV}{SR} = \frac{KV}{TS}$	✓ correct ratios ✓ $\frac{WR}{SR} = \frac{KV}{SV}$ ✓ R ✓ equating $WR \times SV$ (4) ✓ identifying correct Δs ✓ proving $\mid\mid\mid$ ✓ correct ratio ✓ S (4)
		[25]

TOTAL/*TOTAAL*: 150