basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE/ *NASIONALE SENIOR SERTIFIKAAT*

GRADE 12/*GRAAD 12*

MATHEMATICS P1/*WISKUNDE V1*

NOVEMBER 2019

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/*PUNTE*: 150

These marking guidelines consist of 18 pages.
Hierdie nasienriglyne bestaan uit* 18 *bladsye.

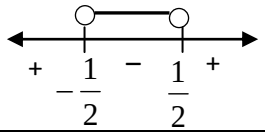
NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- Consistent Accuracy applies in all aspects of the marking memorandum.

LET WEL:

- *Indien 'n kandidaat 'n vraag TWEE keer beantwoord, merk slegs die EERSTE poging.*
- *Volgehoue akkuraatheid is op ALLE aspekte van die nasienriglyne van toepassing.*

QUESTION/*VRAAG* 1

1.1.1	$x^2+5x-6=0$ $(x+6)(x-1)=0$ $x=-6$ or $x=1$	✓factors ✓ $x=-6$ ✓ $x=1$ (3)
1.1.2	$4x^2+3x-5=0$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $x=\frac{-3\pm\sqrt{(3)^2-4(4)(-5)}}{2(4)}$ $x=\frac{-3\pm\sqrt{89}}{8}$ $x=-1{,}55$ or $x=0{,}8$	✓substitution into the correct formula ✓ $x=-1{,}55$ ✓ $x=0{,}8$ (3)
1.1.3	$4x^2-1<0$ $(2x+1)(2x-1)<0$ $\frac{-1}{2}<x<\frac{1}{2}$ 	✓factors ✓method ✓answer (3)
1.1.4	$\left(\sqrt{\sqrt{32}+x}\right)\left(\sqrt{\sqrt{32}-x}\right)=x$ $\sqrt{32-x^2}=x$ $32-x^2=x^2$ $-2x^2=-32$ $x^2=16$ $x=\pm4$ $\therefore x=4$	✓ $\sqrt{32-x^2}$ ✓ squaring both sides ✓ $x^2=16$ ✓ $x=4$ (selection) (4)

1.2	$y+x=12$ $y=-x+12$.....(1) $xy=14-3x$.....(2) Sub (1) into (2) $x(-x+12)=14-3x$ $-x^2+12x-14+3x=0$ $-x^2+15x-14=0$ $x^2-15x+14=0$ $(x-14)(x-1)=0$ $x=14$ or $x=1$ $y=-2$ or $y=11$ **OR/OF** $y+x=12$ $x=-y+12$.....(1) $xy=14-3x$.....(2) Sub (1) into (2) $y(-y+12)=14-3(-y+12)$ $12y-y^2-14+36-3y=0$ $-y^2+9y+22=0$ $y^2-9y-22=0$ $(y+2)(y-11)=0$ $y=-2$ or $y=11$ $x=14$ or $x=1$	✓y subject of the formula ✓ substitution ✓simplification ✓both values of x ✓both values of y (5) **OR/OF** ✓x subject of the formula ✓substitution ✓simplification ✓both values of y ✓both values of x (5)
1.3	3 6 9 12 15 18 21 24 27 30 3 3 3^2 3 3 3^2 3 3 3^3 3 $\therefore k=14$	✓identifying multiples of 3 ✓ten multiples of 3 ✓ powers of 3 ✓ answer (4)
		[22]

QUESTION/*VRAAG* 2

2.1.1	209 ; 186	✓209 ✓186 (2)
2.1.2	321 ; 290 ; 261 ; 234 1*st diff* −31 −29 −27 2*nd diff* 2 2 $2a = 2$ $3a + b = -31$ $a + b + c = 321$ $a = 1$ $3(1) + b = -31$ $1 + (-34) + c = 321$ $b = -34$ $c = 354$ $T_n = n^2 - 34n + 354$	✓ 2nd diff = 2 ✓ $a = 1$ ✓ $b = -34$ ✓ $c = 354$ (4)
2.1.3	$n^2 - 34n + 354 = 74$ $n^2 - 34n + 280 = 0$ $(n-14)(n-20) = 0$ $n = 14$ or $n = 20$	✓equating T_n to 74 ✓standard form ✓14 ✓ 20 (4)
2.1.4	$f'(n) = 0$ $2n - 34 = 0$ $2n = 34$ $n = 17$ Term 17 will have the smallest value **OR/OF** $n = \frac{-b}{2a}$ $n = \frac{34}{2}$ $n = 17$ Term 17 will have the smallest value **OR/OF** $n = \frac{14 + 20}{2} = 17$ Term 17 will have the smallest value	✓ $2n - 34 = 0$ ✓answer (2) **OR/OF** ✓substitution ✓answer (2) **OR/OF** ✓substitution ✓answer (2)

2.2.1	$a=\frac{5}{8}\quad;\quad r=\frac{1}{2}\quad;\quad n=21$ $S_n=\frac{a(1-r^n)}{1-r}$ $S_{21}=\frac{\frac{5}{8}\left(1-\left(\frac{1}{2}\right)^{21}\right)}{1-\frac{1}{2}}$ $=1{,}2499...$ $=1{,}25$	✓ r ✓ substitution into the correct formula ✓ answer (3)
2.2.2	$T_n>\frac{5}{8192}$ $ar^{n-1}>\frac{5}{8192}$ $\frac{5}{8}\left(\frac{1}{2}\right)^{n-1}>\frac{5}{8192}$ $\left(\frac{1}{2}\right)^{n-1}>\frac{1}{1024}$ $\left(\frac{1}{2}\right)^{n-1}>\left(\frac{1}{2}\right)^{10}$ *or* $2^{-n+1}>2^{-10}$ $\therefore n-1<10$ $\quad -n+1>-10$ $n<11$ $\quad n<11$ $\therefore n=10$ $\quad \therefore n=10$ **OR/OF** 8 ; 16 ; 32 ; … ; 8192 $8.2^{n-1}<8192$ $2^{n-1}<1024$ $2^{n-1}<2^{10}$ $n-1<10$ $n<11$ $\therefore n=10$	✓ substitution into the correct formula ✓ method /same base or log ✓calculating n ✓ answer (4) **OR/OF** ✓ substitution into the correct formula ✓ method ✓calculating n ✓ answer (4)
		[19]

QUESTION/*VRAAG* 3

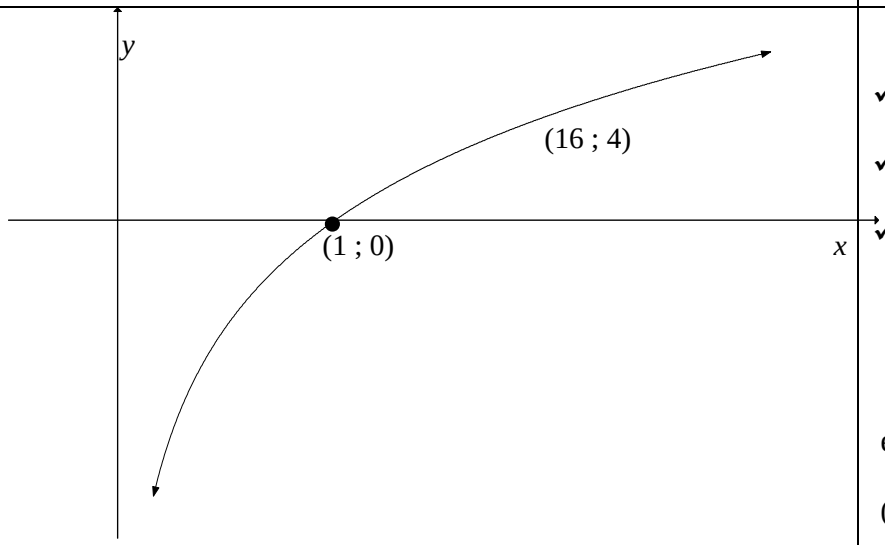
3.1	$\sum_{y=3}^{10}\frac{1}{y-2}-\sum_{y=3}^{10}\frac{1}{y-1}$ $=\left(\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+....+\frac{1}{8}\right)-\left(\frac{1}{2}+\frac{1}{3}+....+\frac{1}{8}+\frac{1}{9}\right)$ $=1-\frac{1}{9}$ $=\frac{8}{9}$	✓ $\left(\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+....+\frac{1}{8}\right)$ ✓$\left(\frac{1}{2}+\frac{1}{3}+....+\frac{1}{8}+\frac{1}{9}\right)$ ✓answer (3)
3.2	$\left(\frac{1}{3}\times\frac{2}{3}\right)+\left(\frac{2}{3}\times\frac{2}{3}\right)+\left(1\times\frac{2}{3}\right)+....+\left(4\times\frac{2}{3}\right)$ $=\frac{2}{9}+\frac{4}{9}+\frac{2}{3}+....+\frac{8}{3}$ $a=\frac{2}{9}$ and $d=\frac{2}{3}-\frac{4}{9}=\frac{2}{9}$ $S_n=\frac{n}{2}[2a+(n-1)d]$ OR $S_n=\frac{n}{2}(a+l)$ $S_{12}=\frac{12}{2}\left[2\left(\frac{2}{9}\right)+(12-1)\frac{2}{9}\right]$ $S_{12}=\frac{12}{2}\left(\frac{2}{9}+\frac{8}{3}\right)$ $=\frac{52}{3}\text{m}^2$ $=\frac{52}{3}\text{m}^2$ $\therefore$ for both sides $=2\times\frac{52}{3}=\frac{104}{3}=34{,}67\text{m}^2$ **OR/OF** $\frac{2}{9}\times(1+2+3+4+5+6+7+8+9+10+11+12)\times 2$ $=34{,}67\text{ m}^2$ **OR/OF** $T_1=\frac{2}{9}\times 12=\frac{8}{3}$ $l=\frac{2}{9}\times 1=\frac{2}{9}$ $2S_{12}=2\left(\frac{12}{2}\right)\left(\frac{8}{3}+\frac{2}{9}\right)$ $=34{,}67\text{ m}^2$	✓✓a ✓d ✓ substitution into the correct formula ✓ answer ✓ answer for both sides (6) **OR/OF** ✓✓ a ✓✓ $(1 + ... + 12)$ ✓ $\times 2$ ✓ answer (6) **OR/OF** ✓✓ a ✓ $T_1=\frac{8}{3}$✓ $l=\frac{2}{9}$ ✓ substitution into correct formula ✓ answer (6)
		[9]

QUESTION/*VRAAG* 4

4.1	$p=-1$	✓ $p=-1$ (1)
4.2	$y=\frac{a}{x-1}$ $-3=\frac{a}{0-1}$ $a=3$ $y=x^2+bx-3$ $0=(1)^2+(1)b-3$ $b=2$	✓coordinates D(0 ; –3) ✓substitute (0 ; –3) ✓substitute (1 ; 0) (3)
4.3	$y=x^2+2x-3$ axis of sym: $x=\frac{-b}{2a}$ $x=\frac{-2}{2(1)}$ $x=-1$ $y=(-1)^2+2(-1)-3=-4$ C$(-1;-4)$ **OR/OF** $\frac{dy}{dx}=0$ $2x+2=0$ $x=-1$ $y=(-1)^2+2(-1)-3=-4$ C$(-1;-4)$	✓substitution ✓ $x=-1$ ✓substitution ✓$y=-4$ (4) **OR/OF** ✓derivative ✓ $x=-1$ ✓substitution ✓$y=-4$ (4)
4.4	$y\in[-4;\infty)$ or $y\geq -4$	✓ -4 ✓ answer (2)
4.5	$m=\tan 45°=1$ $y=mx+c$ $-4=(1)(-1)+c$ $c=-3$ $y=x-3$	✓gradient ✓subs m and $(-1;-4)$ ✓equation (3)
4.6	No, the line passes through C and D **OR/OF** No, a tangent through turning point C will have a gradient of 0	✓ No ✓ reason (2) **OR/OF** ✓ No ✓ reason (2)

4.7	$f(m-x)=f[-(x-m)]$ f is reflected in the y-axis and translated 1 unit to the left and 4 units upwards. Therefore: $m=-1$ $q=4$ **OR/OF** Substitute $x=0$ and $q=4$ for one x- intercept $h(x)=(m-x)^2+2(m-x)-3+q$ $h(0)=(m-0)^2+2(m-0)-3+4$ $0=m^2+2m+1$ $0=(m+1)^2$ $m=-1$ $q=4$	✓✓value of m ✓✓ value of q (4) **OR/OF** ✓✓value of m ✓✓ value of q (4)
		[19]

QUESTION/*VRAAG* 5

5.1	$f(x)=k^x$ $16=k^4$ $k=2$	✓substitution (4 ; 16) ✓answer (2)
5.2	$f:\quad y=2^x$ $f^{-1}:\quad x=2^y$ $y=\log_2 x$	✓ $x=2^y$ ✓ $y=\log_2 x$ (2)
5.3		✓asymptote ✓shape ✓✓for any two valid points eg.(16 ; 4) or (2 ; 1) or (4 ; 2) or (1 ; 0) (4)
5.4.1	$x\in(1;\infty)$ or $x>1$	✓1 ✓ answer (2)
5.4.2	$0<x\leq\frac{1}{2}$ or $x\in\left(0;\frac{1}{2}\right]$	✓$\frac{1}{2}$ ✓ answer (2)

5.5	$2^x - 2^{-x} = \frac{15}{4}$ $2^x - \frac{1}{2^x} = \frac{15}{4}$ $2^{2x} - 1 = \frac{15}{4} \times 2^x$ $4.2^{2x} - 4 = 15 \times 2^x$ $4.2^{2x} - 15.2^x - 4 = 0$ $(4.2^x + 1)(2^x - 4) = 0$ $4.2^x + 1 = 0$ or $2^x - 4 = 0$ $2^x = \frac{-1}{4}$ or $2^x = 2^2$ N/A $x = 2$	✓ $2^x - 2^{-x} = \frac{15}{4}$ ✓standard form ✓factors ✓answer (4)
	OR/OF $2^x - 2^{-x} = \frac{15}{4}$ $2^x - \frac{1}{2^x} = \frac{15}{4}$ Let $k = 2^x$ $k^2 - 1 = \frac{15}{4} \times k$ $4.k^2 - 4 = 15 \times k$ $4.k^2 - 15.k - 4 = 0$ $(4.k + 1)(k - 4) = 0$ $k = \frac{-1}{4}$ or $k = 4$ $2^x = \frac{-1}{4}$ or $2^x = 2^2$ N/A $x = 2$	**OR/OF** ✓ $2^x - 2^{-x} = \frac{15}{4}$ ✓standard form ✓factors ✓answer (4)
		[16]

QUESTION/*VRAAG* 6

6.1	Kuda : $A = P(1+in)$ $= 5\ 000(1+0{,}083\times 4)$ $= R6\ 660{,}00$ Final Answer : R6 660,00 + R266,40 $= R6\ 926{,}40$ **OR/OF** Kuda : $A = P(1+in)\times 1{,}04$ $= 5\ 000(1+0{,}083\times 4)\times 1{,}04$ $= R6\ 926{,}40$ Thabo : $A = P(1+i)^n$ $= 5\ 000\left(1+\frac{0{,}081}{12}\right)^{12\times 4}$ $= R6\ 905{,}71$ Kuda will have a better investment	✓substitution into the correct formula ✓final answer **OR/OF** ✓substitution into the correct formula ✓final answer ✓substitution into the correct formula ✓answer ✓conclusion (5)
6.2.1	$P = \frac{x[1-(1+i)^{-n}]}{i}$ $525\ 000 = \frac{6\ 000\left[1-\left(1+\frac{0{,}1}{12}\right)^{-n}\right]}{\frac{0{,}1}{12}}$ $\frac{35}{48} = 1-\left(1+\frac{0{,}1}{12}\right)^{-n}$ $-n\log\left(1+\frac{0{,}1}{12}\right) = \log\frac{13}{48}$ $-n = \frac{\log\frac{13}{48}}{\log\left(1+\frac{0{,}1}{12}\right)}$ $n = 157{,}40$ n = 158 payments **OR/OF**	✓$\frac{0{,}1}{12}$ ✓substitution into the correct formula ✓simplification ✓use of logs ✓answer (5) **OR/OF**

	$P = \frac{x[1-(1+i)^{-n}]}{i}$ $525\,000 = \frac{6\,000\left[1-\left(1+\frac{0,1}{12}\right)^{-12n}\right]}{\frac{0,1}{12}}$ $\frac{35}{48} = 1-\left(1+\frac{0,1}{12}\right)^{-12n}$ $-12n\log\left(1+\frac{0,1}{12}\right) = \log\frac{13}{48}$ $-12n = \frac{\log\frac{13}{48}}{\log\left(1+\frac{0,1}{12}\right)}$ $n = \frac{\log\frac{13}{48}}{\log\left(1+\frac{0,1}{12}\right)} \times \frac{1}{12}$ $n = 13,11686841$ Number of payments = $13,11686841 \times 12 = 157,40$ n = 158 payments	✓ $\frac{0,1}{12}$ ✓substitution into the correct formula ✓simplification ✓use of logs ✓answer (5)
6.2.2	Difference: R6 000 – R5 066,36 = R933,64 $F = \frac{x[(1+i)^n - 1]}{i}$ $F = \frac{933,64\left[\left(1+\frac{0,1}{12}\right)^{108} - 1\right]}{\frac{0,1}{12}}$ $= R162\,503,51$ **OR/OF**	✓ R933,64 ✓ n = 108 ✓ substitution into the correct formula ✓ answer (4) **OR/OF**

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$F = \frac{6\,000\left[\left(1+\frac{0,1}{12}\right)^{108} - 1\right]}{\frac{0,1}{12}}$$

$= \text{R}1\,044\,322,28$

✓ n = 108
✓substitution into correct formula

$$F = \frac{5\,066,36\left[\left(1+\frac{0,1}{12}\right)^{108} - 1\right]}{\frac{0,1}{12}}$$

$F = \text{R}881\,818,77.....$

✓ substitution into correct formula

Amount available for withdrawal
= R1 044 322,28 – R 881 818,77
= R162 503,51

✓final answer

(4)

OR/OF

OR/OF

Outstanding balance with monthly repayment of R5 066,35

$$= 525\,000\left(1+\frac{0,1}{12}\right)^{108} - \frac{5\,066,36\left[\left(1+\frac{0,1}{12}\right)^{108} - 1\right]}{\frac{0,1}{12}}$$

= R404 666, 23

✓n = 108
✓substitution into the correct formula

Outstanding balance with monthly repayment of R6 000

$$= 525\,000\left(1+\frac{0,1}{12}\right)^{108} - \frac{6\,000\left[\left(1+\frac{0,1}{12}\right)^{108} - 1\right]}{\frac{0,1}{12}}$$

= R242 162,72

✓ substitution into the correct formula

Amount available for withdrawal
R404 666,23 – R242 162,72 = R162 512,18

✓final answer

(4)

[14]

QUESTION/*VRAAG* 7

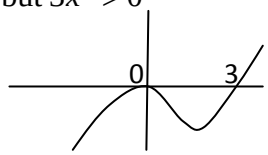
7.1	$f(x)=4-7x$ $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $=\lim\limits_{h\to 0}\dfrac{4-7(x+h)-(4-7x)}{h}$ $=\lim\limits_{h\to 0}\dfrac{h(-7)}{h}$ $=-7$	✓ $4-7(x+h)$ ✓substitution ✓simplification ✓answer (4)
7.2	$y=4x^8+\sqrt{x^3}$ $=4x^8+x^{\frac{3}{2}}$ $\dfrac{dy}{dx}=32x^7+\dfrac{3}{2}x^{\frac{1}{2}}$	✓ $x^{\frac{3}{2}}$ ✓ $32x^7$ ✓ $\dfrac{3}{2}x^{\frac{1}{2}}$ (3)
7.3.1	$y=ax^2+a$ $\dfrac{dy}{dx}=2ax+0$ $\dfrac{dy}{dx}=2ax$	✓ $2ax$ (1)
7.3.2	$y=ax^2+a$ $\dfrac{dy}{da}=x^2+1$	✓ ✓ answer (2)

7.4	Substitute $(2\ ;\ b)$ in $y = x + \frac{12}{x}$ $b = 2 + \frac{12}{2}$ $b = 8$ $m_{\text{tangent}} = \frac{dy}{dx}$ $\frac{dy}{dx} = 1 - \frac{12}{x^2}$ $m_{\text{tangent}} = 1 - \frac{12}{2^2} = -2$ $m_{\text{perp}} = \frac{1}{2}$ Equation of perpendicular line: $y - y_1 = m(x - x_1)$ OR $y = mx + c$ $y - 8 = \frac{1}{2}(x - 2)$ $8 = \frac{1}{2}(2) + c$ $y = \frac{1}{2}x + 7$ $c = 7$ $y = \frac{1}{2}x + 7$	✓ value of b ✓ $\frac{dy}{dx} = 1 - \frac{12}{x^2}$ ✓gradient of perpendicular line ✓ equation (4)
		[14]

QUESTION/*VRAAG* 8

8.1	36cm	✓answer (1)
8.2	$\therefore t = 6$ $(-2t^2 + 3t - 6)$ have no real roots Insect reaches the floor only once.	✓✓✓ only once (3)
8.3	$h(t) = -2t^3 + 15t^2 - 24t + 36$ $h'(t) = -6t^2 + 30t - 24$ $-6t^2 + 30t - 24 = 0$ $t^2 - 5t + 4 = 0$ $(t - 4)(t - 1) = 0$ $t = 4$ or $t = 1$ Only $t = 4$ because maximum value required $h = -2(4)^3 + 15(4)^2 - 24(4) + 36 = 52\ cm$	✓ expansion ✓ $-6t^2 + 30t - 24 = 0$ ✓both values ✓answer (4)
		[8]

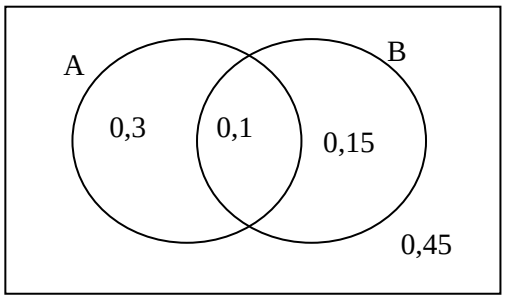
QUESTION/*VRAAG* 9

9.1	$f'(x) = 9x^2$ $3x^3 = 9x^2$ $3x^3 - 9x^2 = 0$ $3x^2(x-3) = 0$ $x = 0$ or $x = 3$	✓ $f'(x) = 9x^2$ ✓ $x = 0$ ✓ $x = 3$ (3)
9.2.1	For f and f'	✓ answer (1)
9.2.2	The point (0 ; 0) is : A point of inflection of f A turning point of f'	✓ f : inflection point ✓ f' : turning point (2)
9.3	$f''(x) = 18x$ Distance $= f''(1) - f'(1)$ $= 18(1) - 9(1)^2$ $= 9$	✓ $f''(x) = 18x$ ✓substitution ✓answer (3)
9.4	$3x^3 - 9x^2 < 0$ $3x^2(x-3) < 0$ but $3x^2 > 0$  $\therefore x - 3 < 0$ $\therefore x < 3$, $x \neq 0$	✓ $3x^3 - 9x^2 < 0$ ✓ factors ✓ $x < 3$ ✓ $x \neq 0$ (4)
		[13]

QUESTION/*VRAAG* 10

10.1	P(same day) $= \frac{4}{16}$ or $\frac{1}{4}$ or 0,25 or 25%	✓4 numerator ✓16 denominator (2)
10.2	P(2 consecutive days) $= \frac{3\times 2}{16} = \frac{3}{8}$	✓3 ✓$\times$2 ✓ answer (3)
		[5]

QUESTION/*VRAAG* 11

11.1.1	$P(A)\times P(B)$ independent events $=0,40\times 0,25 \ =0,1$  	✓0,1 ✓0,15 and 0,3 ✓0,45 (3)
11.1.2	$P(\text{A or not B}) = P(\text{A}) + P(\text{not B}) - P(\text{A and not B})$ $= 0,4 + 0,75 - 0,3$ $= 0,85$ **OR/OF** $P(\text{A or not B}) = 1 - P(\text{only B})$ $= 1 - 0,15$ $= 0,85$ **OR/OF** From Venn diagram: 0,3 + 0,1 + 0,45 = 0,85	✓ substitution ✓answer (2) **OR/OF** ✓ 1 – 0,15 ✓answer (2) **OR/OF** ✓ substitution ✓ answer (2)
11.2	$(5 \times 1 \times 5) + (5 \times 1 \times 6) + (5 \times 1 \times 6) + (5 \times 1 \times 5) = 110$ 110 × 5 = 550 > 500 Not possible, because not enough space **OR/OF** (5 × 2 × 5) + (5 × 2 × 6) = 110 110 × 5 = 550 > 500 Not possible because not enough space **OR/OF**	✓5 × 1 × 5 ✓5 × 1 × 6 ✓5 × 1 × 6 ✓5 × 1 × 5 ✓110 ✓ conclusion (6) **OR/OF** ✓✓5 × 2 × 5 ✓✓5 × 2 × 6 ✓ 110 ✓ conclusion (6) **OR/OF**

	$5\times4\times6=120$ $5\times2=10$ $\therefore 120-10=110$ $110 \times 5 = 550 > 500$ Not possible because not enough space	✓✓ $5\times4\times6=120$ ✓ $5\times2=10$ ✓ $120-10$ ✓ 110 ✓ conclusion (6)
		[11]

TOTAL/*TOTAAL:* 150